Characterization and Design of Filter Inductors and Capacitors to Suppress the Radiated EMI in A Power Converter

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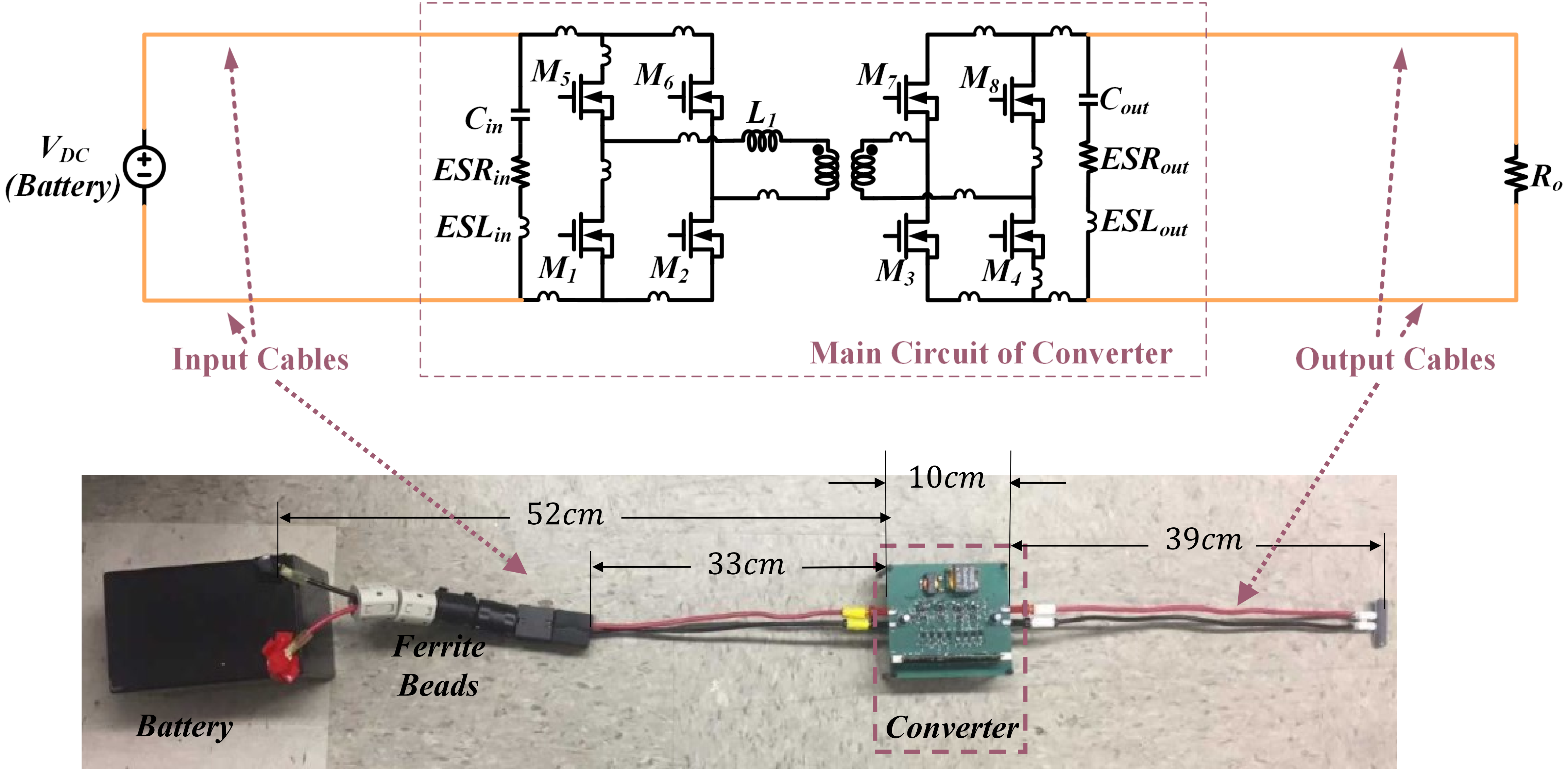
July, 2022

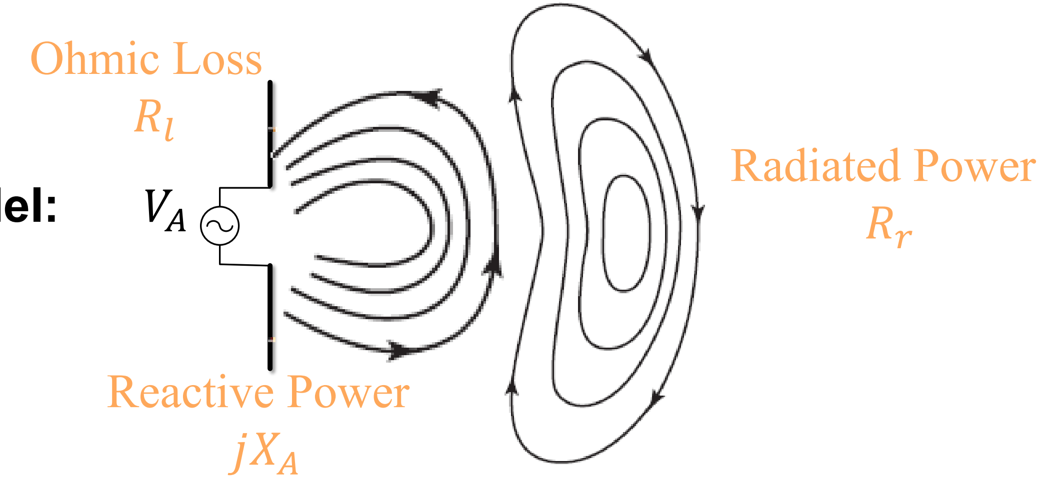
Power Converter's Radiated EMI Model
(A Dual Active Bridge Converter Example)

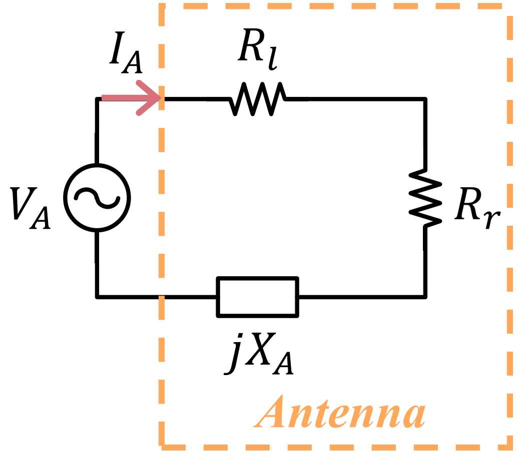


Power Converter's Radiated EMI Model

(A Dual Active Bridge Converter Example)

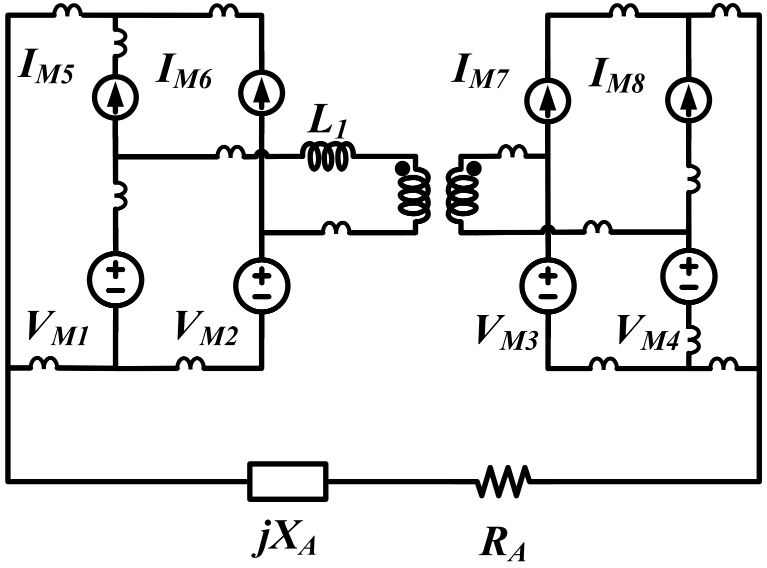
Cable antenna model:

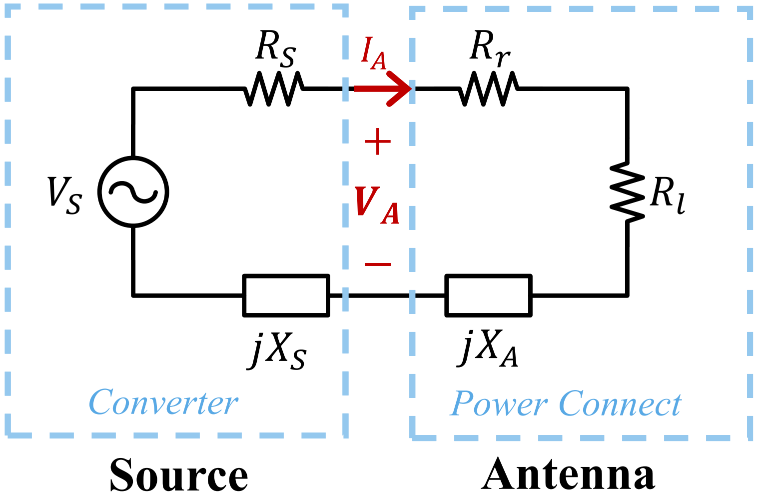




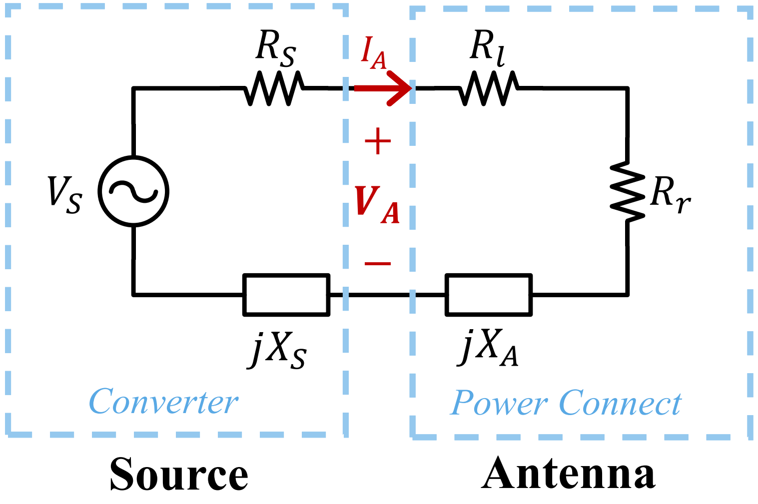
$$R_A = R_l + R_r$$

Power converter radiated EMI model:





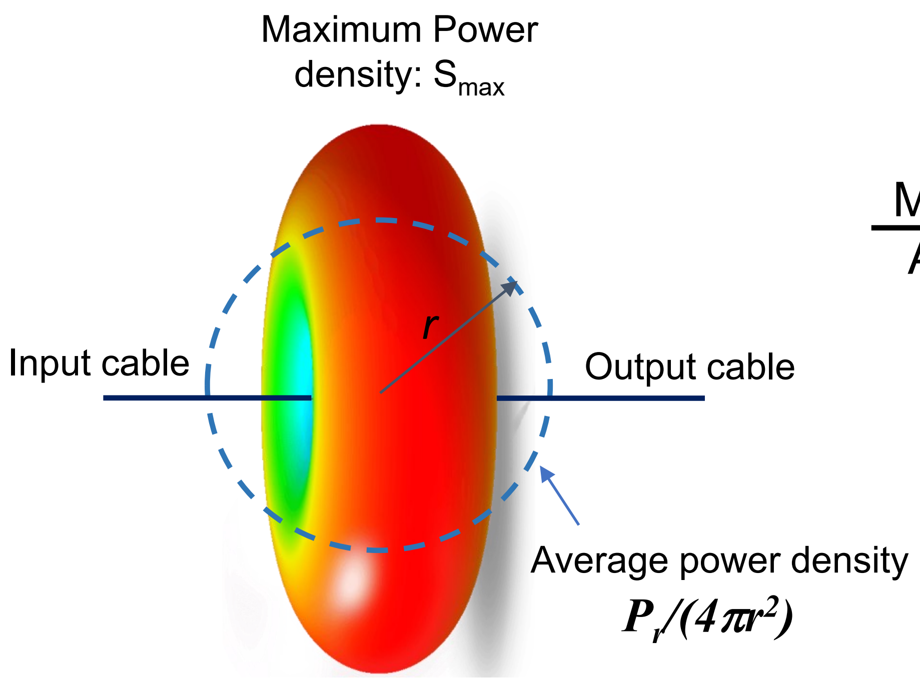
The Radiated Electric Field



Radiated Power:

$$P_r = \frac{1}{2}|I_A|^2 R_r$$

$$|I_A| = \frac{|V_S|}{\sqrt{(R_S + R_A)^2 + (X_S + X_A)^2}} = |V_S| \times K_I$$



Directivity D:

$$D = \frac{\text{Maximum power density}}{\text{Average power density}}$$

$D = S_{max}(4\pi r^2) / P_r$

$S_{max} = E_{max}^2 / 2\eta$

(η: wave impedance)

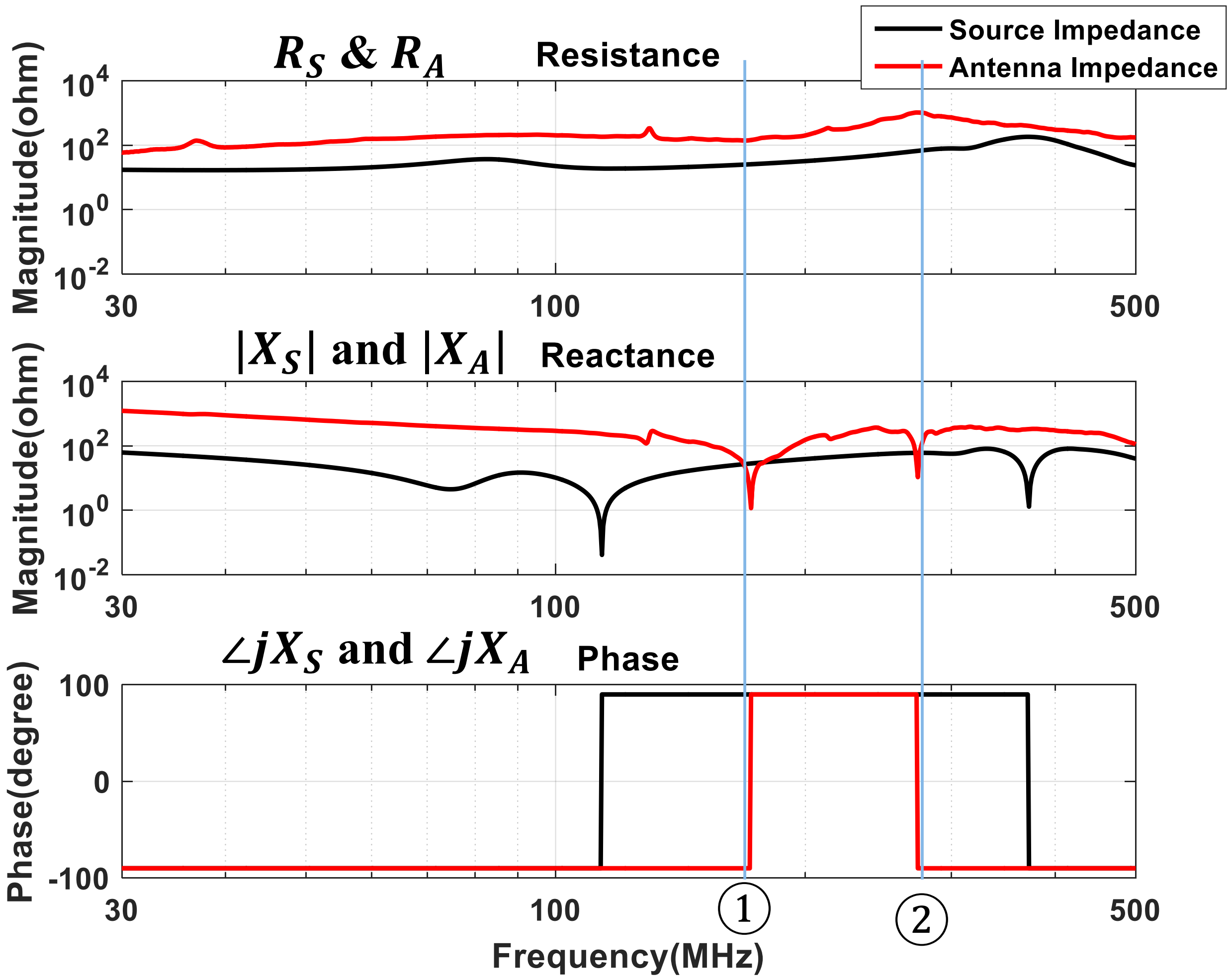
$G_o = eD = DR_r/R_A = DR_r/(R_r + R_l)$

$$E_{max} = \sqrt{\frac{\eta G_o}{4\pi r^2}} \times |V_S| \times \frac{\sqrt{R_A}}{\sqrt{(R_S+R_A)^2+(X_S+X_A)^2}} = \sqrt{\frac{\eta G_o}{4\pi r^2}} \times |V_S| \times K_E$$

$$K_I = \frac{1}{\sqrt{(R_S + R_A)^2 + (X_S + X_A)^2}}$$

$$K_E = \frac{\sqrt{R_A}}{\sqrt{(R_S+R_A)^2+(X_S+X_A)^2}}$$

EMI Spikes due to the Interaction of Antenna and Converter Impedances



$$|I_A| = \frac{|V_S|}{\sqrt{(R_S+R_A)^2+(X_S+X_A)^2}} = |V_S| * K_I$$

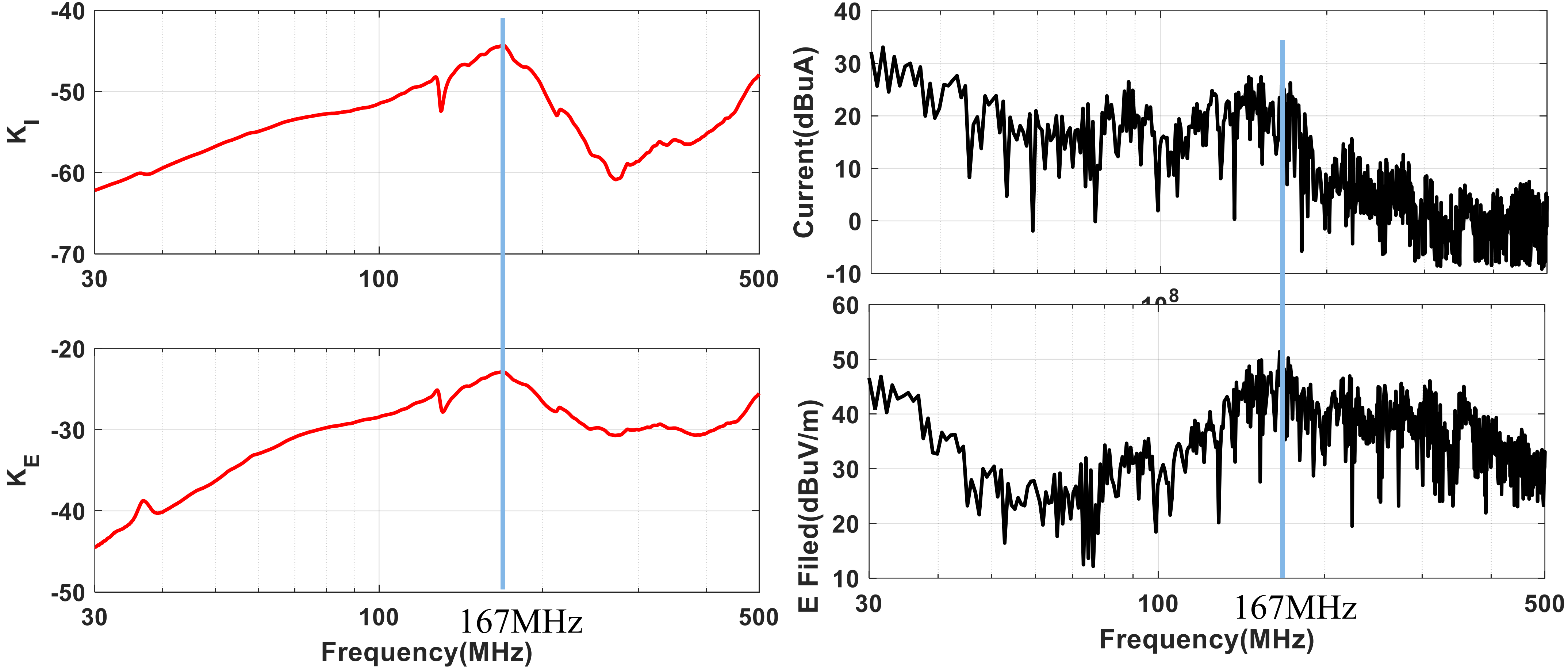
$$E_{max} = \sqrt{\frac{\eta G_o}{4\pi r^2}} \times |V_S| \times \frac{\sqrt{R_A}}{\sqrt{(R_S+R_A)^2+(X_S+X_A)^2}} = \sqrt{\frac{\eta G_o}{4\pi r^2}} \times |V_S| \times K_E$$

$$K_I = \frac{1}{\sqrt{(R_S+R_A)^2+(X_S+X_A)^2}}$$

$$K_E = \frac{\sqrt{R_A}}{\sqrt{(R_S+R_A)^2+(X_S+X_A)^2}}$$

Number	Frequency (MHz)	R_S (Ω)	R_A (Ω)	K_I (S)	K_E ($\sqrt{S}$)
①	167	24	140	0.006	0.072
②	270	68	1030	0.0009	0.029

Current and Radiated Electric Field due to Interactions



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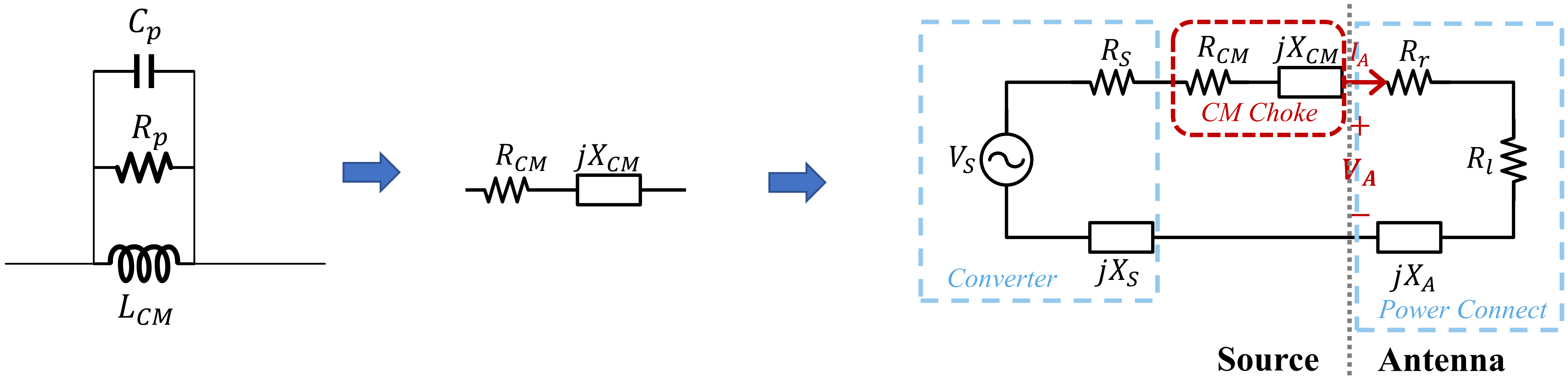
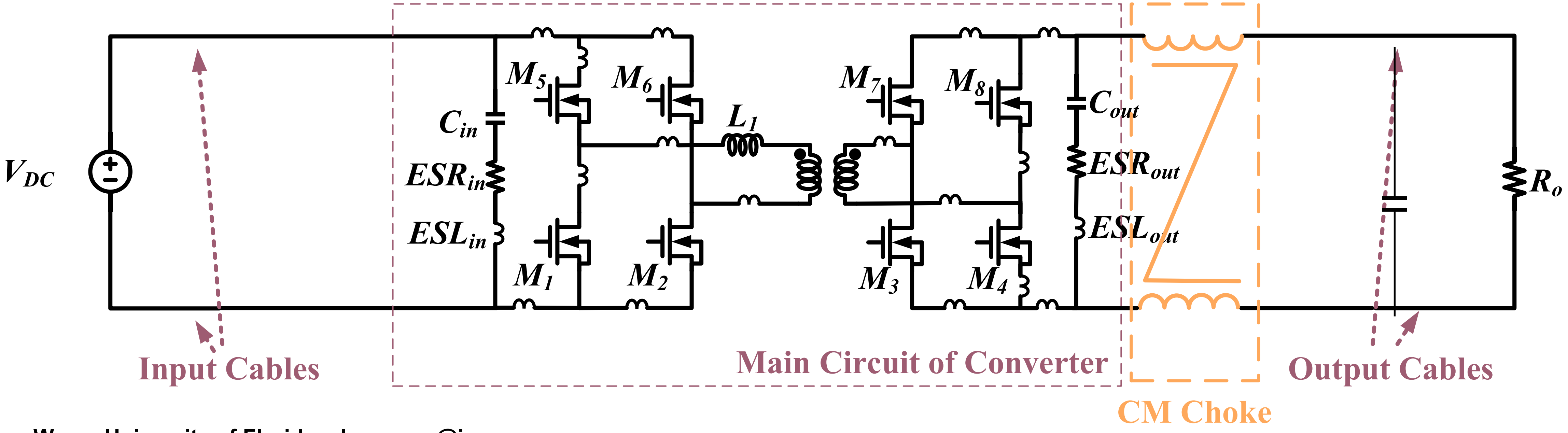
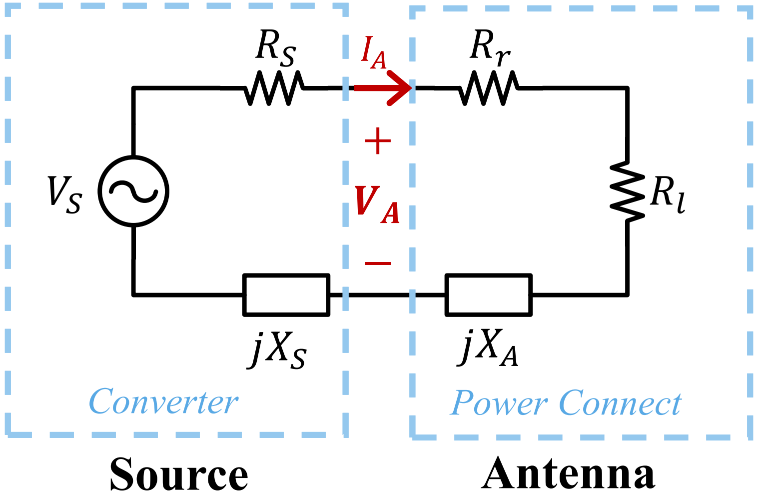
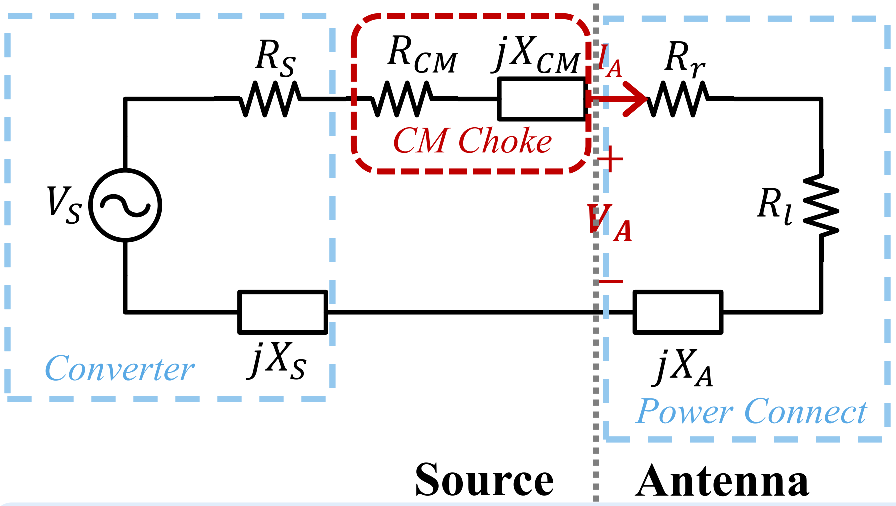
Inductor Design to Reduce Radiated EMI





CM Inductor’s Impact to Radiated EMI





$$|I_A| = \frac{|V_S|}{\sqrt{(R_S + R_A)^2 + (X_S + X_A)^2}}$$

$$E_{max} = \sqrt{\frac{\eta G_o}{4\pi r^2}} \times |V_S| \times \frac{\sqrt{R_A}}{\sqrt{(R_S + R_A)^2 + (X_S + X_A)^2}}$$

$$|I_A| = \frac{|V_S|}{\sqrt{(R_S + \boldsymbol{R_{CM}} + R_A)^2 + (X_S + \boldsymbol{X_{CM}} + X_A)^2}}$$

$$E_{max} = \sqrt{\frac{\eta G_o}{4\pi r^2}} \times |V_S| \times \frac{\sqrt{R_A}}{\sqrt{(R_S + \boldsymbol{R_{CM}} + R_A)^2 + (X_S + \boldsymbol{X_{CM}} + X_A)^2}}$$

$$K_I = \frac{1}{\sqrt{(R_S + R_A)^2 + (X_S + X_A)^2}}$$

$$K_E = \frac{\sqrt{R_A}}{\sqrt{(R_S + R_A)^2 + (X_S + X_A)^2}}$$

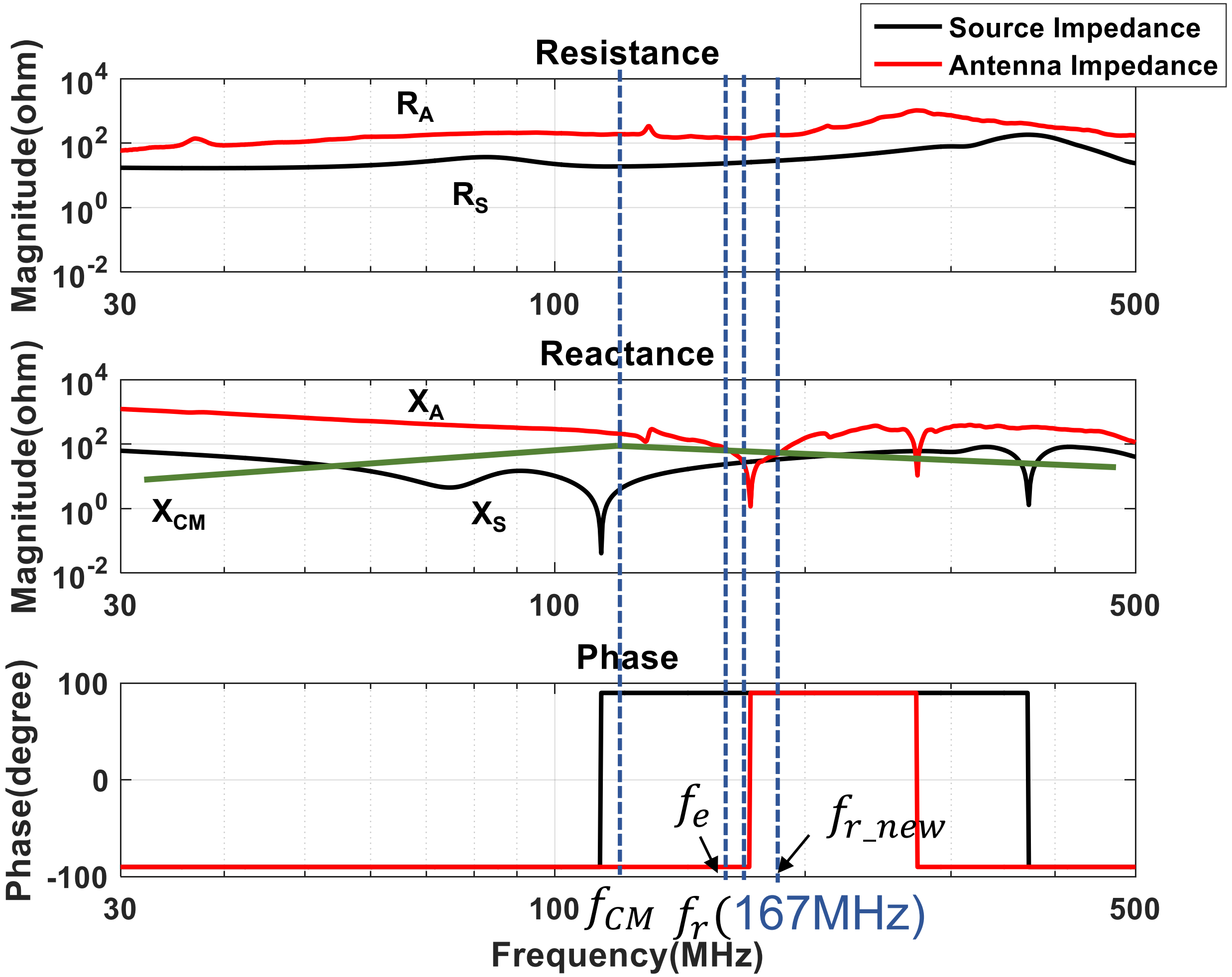
$$K_I = \frac{1}{\sqrt{(R_S + \boldsymbol{R_{CM}} + R_A)^2 + (X_S + \boldsymbol{X_{CM}} + X_A)^2}}$$

$$K_E = \frac{\sqrt{R_A}}{\sqrt{(R_S + \boldsymbol{R_{CM}} + R_A)^2 + (X_S + \boldsymbol{X_{CM}} + X_A)^2}}$$

CM Choke will affect the radiation by
- **Shifting the resonant frequency**
- **Increasing the total resistance**
- **Changing the total reactance**

Design of CM Inductor Reactance

At the resonant frequency f_r, antenna X_A is capacitive, source X_S is inductive



1. If $X_{CM} < 0$ at f_r, to push the new resonant frequency higher becuase source noise spectrum is lower at higher frequenices:

$$|X_{CM}| > |X_S| \ @\ f_r$$

➔ $|X_{CM}| > 35.74@167MHz$ and CM choke is capacitive

➔ $\frac{1}{2\pi * 167M * C_p} > 35.74$ ➔ $C_p < 26.67pF$

➔ to avoid an additional resonant frequency at low f, two conditions should be met:

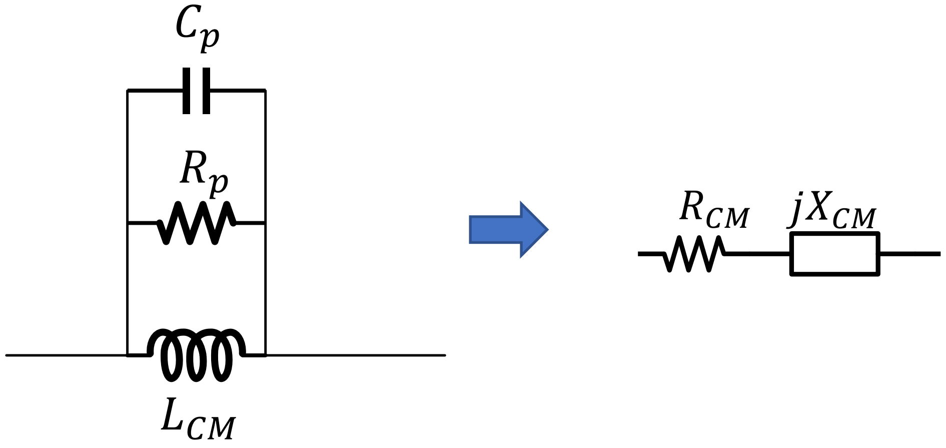
➔ A) $f_{CM} = \frac{1}{2\pi\sqrt{L_{CM}C_p}} < f_e$ *where* f_{CM} is the self-resonant frequency of the CM choke and f_e is the frequency at which $X_A = X_{CM}$

B) $|X_{CM}| < |X_A|$ when $f < f_e$

2. If $X_{CM} > 0$ at f_r

the new resonant frequency will be lower, avoid!

Resistance and Reactance of the CM Inductor



$$R_{CM} = \frac{\frac{\omega^2 {L_{CM}}^2}{R_p}}{\omega^4 {L_{CM}}^2 {C_p}^2 + \omega^2 {L_{CM}}^2 \left(\frac{1}{{R_p}^2} - 2C_p\right) + 1}$$

$$jX_{CM} = j\frac{\omega L_{CM}(1 - \omega^2 L_{CM} C_p)}{\omega^4 {L_{CM}}^2 {C_p}^2 + \omega^2 {L_{CM}}^2 \left(\frac{1}{{R_p}^2} - 2C_p\right) + 1}$$

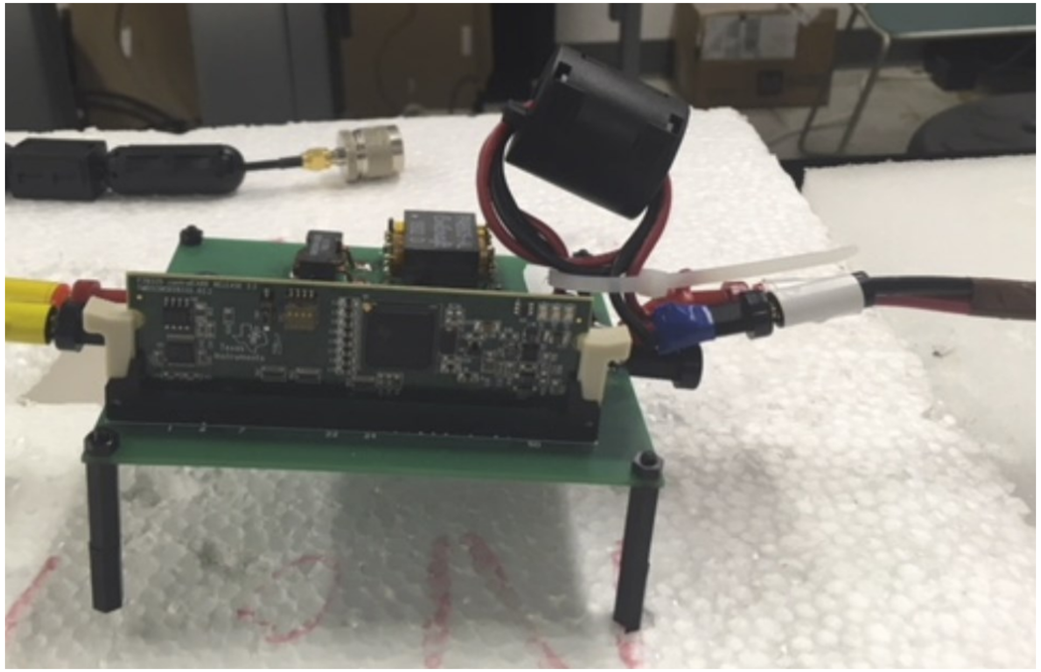
$$f_{CM} = \frac{1}{2\pi\sqrt{L_{CM} C_p}}$$

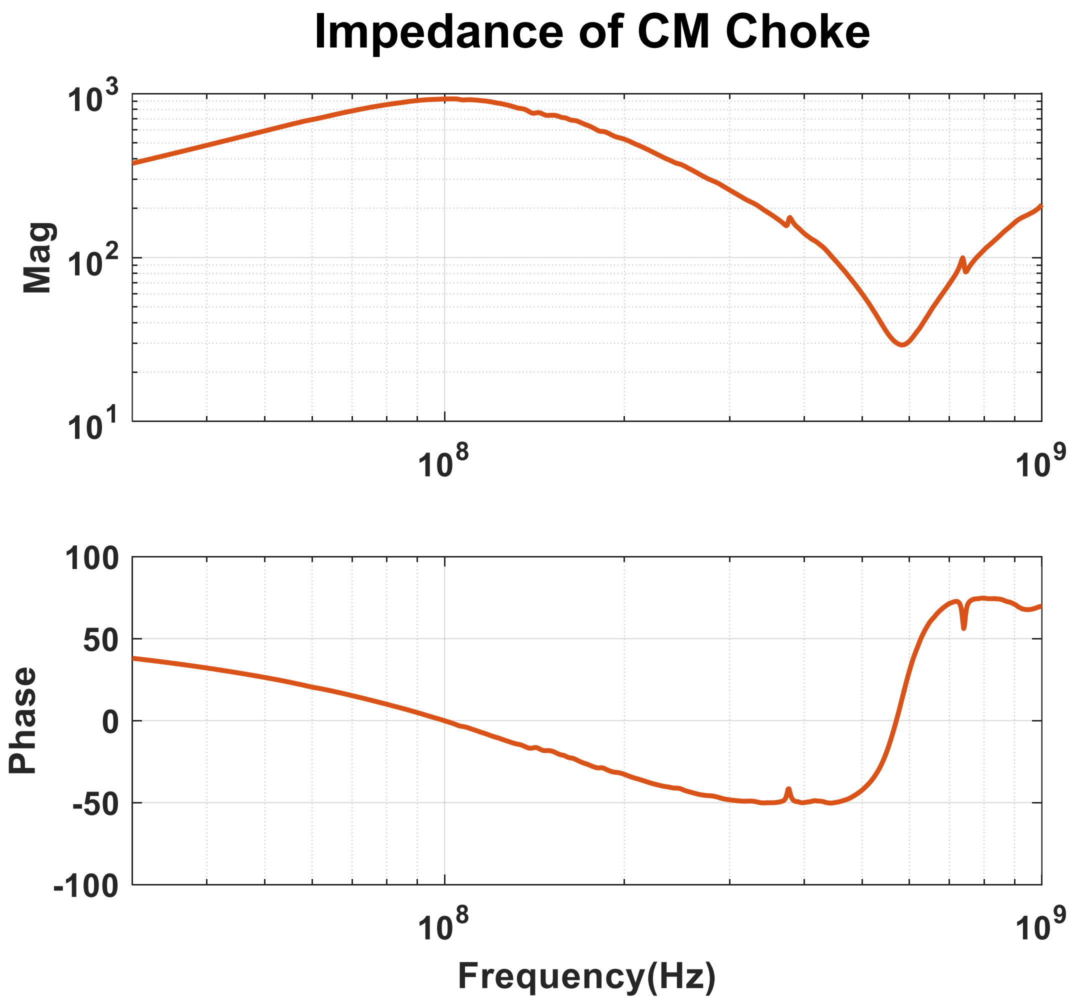
$$R_{CM} = \begin{cases} \frac{\omega^2 {L_{CM}}^2}{R_p} & f \ll f_{CM} \\ R_p & f = f_{CM} \\ \frac{1}{\omega^2 {C_p}^2 R_p} & f \gg f_{CM} \end{cases}$$

R_{CM} is the maximum at f_{CM}
➔
f_{CM} is better to be close to f_{rnew}

$$X_{CM} = \begin{cases} \omega L_{CM} & f \ll f_{CM} \\ 0 & f = f_{CM} \\ \frac{1}{\omega C_p} & f \gg f_{CM} \end{cases}$$

CM Inductor with the Desired Impedance





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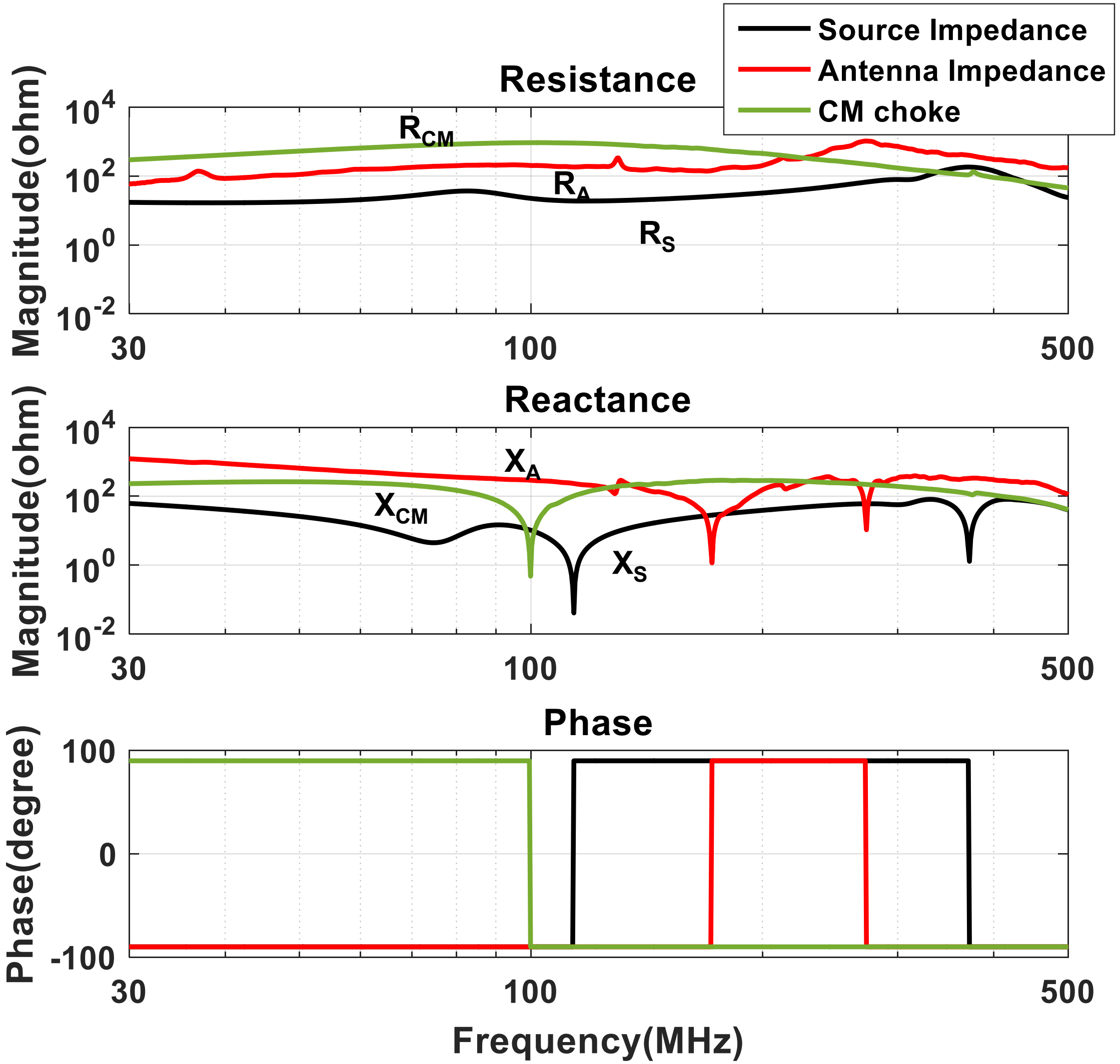
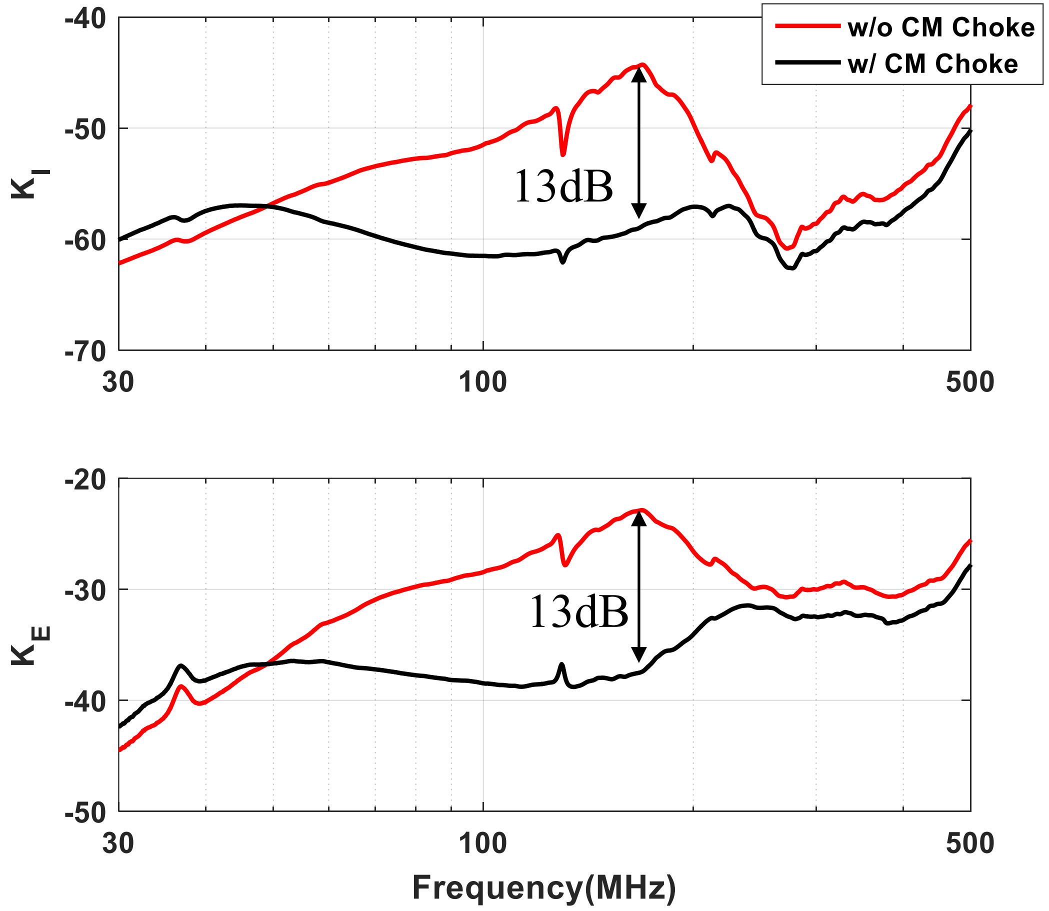
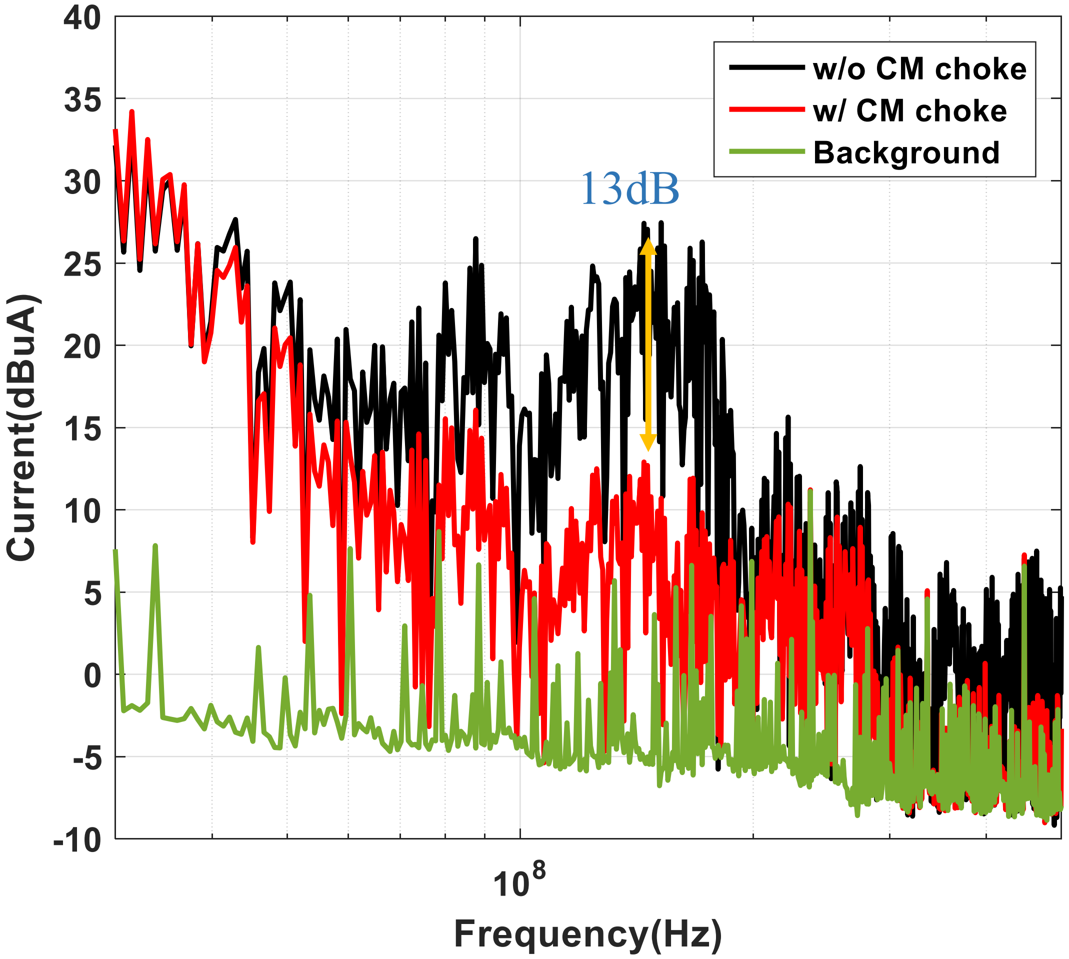
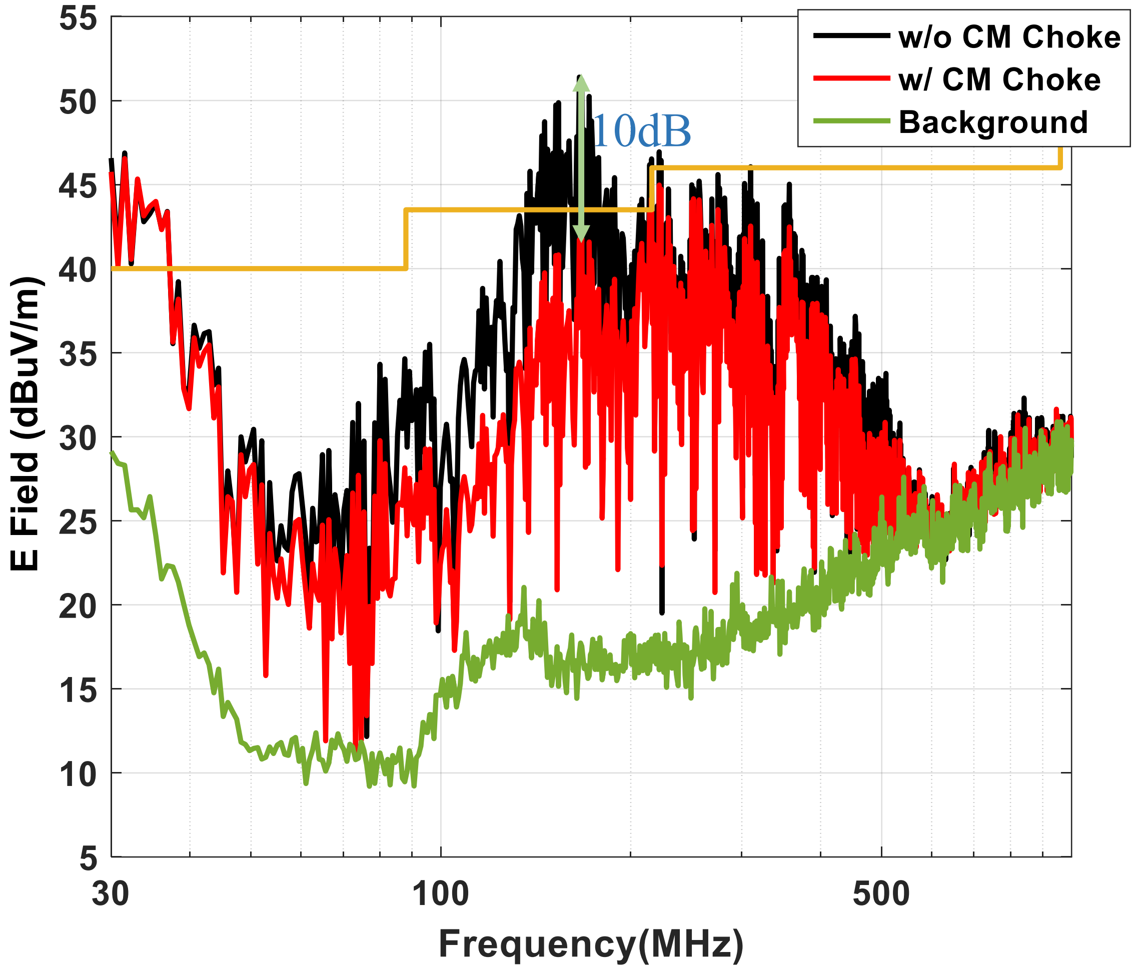
Radiated EMI Reduction w/ the Redesigned CM Inductor



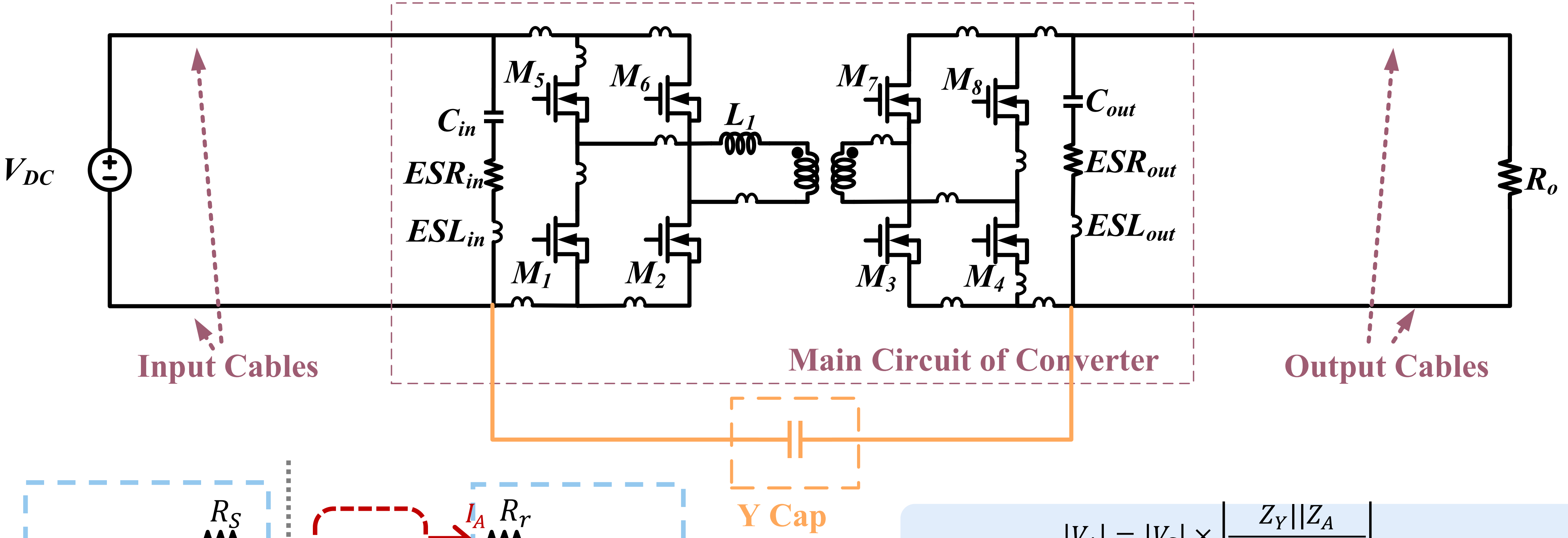


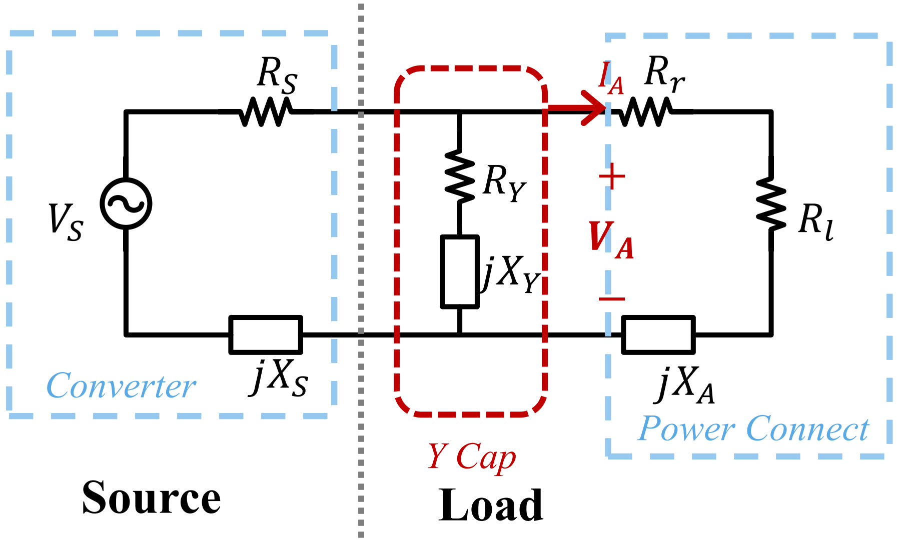
Experimental Verification





CM Capacitor Design to Reduce Radiated EMI





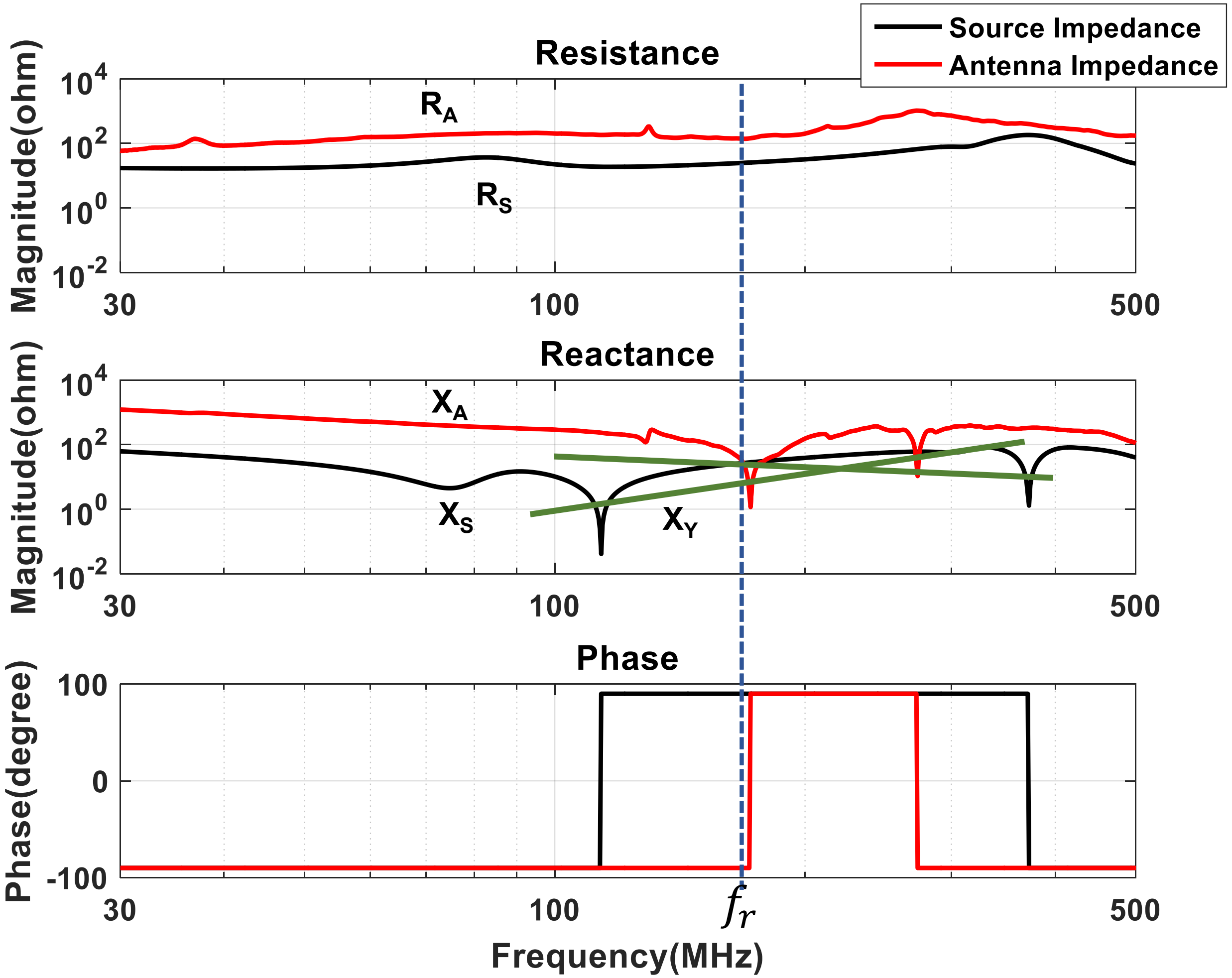
$$|V_A| = |V_S| \times \left|\frac{Z_Y||Z_A}{Z_Y||Z_A + Z_S}\right|$$

$$|I_A| = |V_S| \times \left|\frac{Z_Y||Z_A}{Z_Y||Z_A + Z_S}\right| \times \frac{1}{|Z_A|}$$

$$E_{max} = \sqrt{\frac{\eta G_o}{4\pi r^2}} \times |V_S| \times \left|\frac{Z_Y||Z_A}{Z_Y||Z_A + Z_S}\right| \times \frac{\sqrt{R_A}}{|Z_A|}$$

$$Z_Y = R_Y + jX_Y$$
$$Z_A = R_A + jX_A$$
$$Z_S = R_S + jX_S$$

Impact of CM Capacitor on Radiated EMI



$$E_{max} = \sqrt{\frac{\eta G_o}{4\pi r^2}} \times |V_S| \times \left|\frac{Z_Y||Z_A}{Z_Y||Z_A + Z_S}\right| \times \frac{\sqrt{R_A}}{|Z_A|}$$

1. If the Y cap is effective ➔ $|Z_Y| < |Z_A|$

$$E_{max} = \sqrt{\frac{\eta G_o}{4\pi r^2}} \times |V_S| \times \left|\frac{Z_Y||Z_A}{Z_Y||Z_A + Z_S}\right| \times \frac{\sqrt{R_A}}{|Z_A|}$$
$$= \sqrt{\frac{\eta G_o}{4\pi r^2}} \times |V_S| \times \left|\frac{Z_Y}{Z_Y + Z_S}\right| \times \frac{\sqrt{R_A}}{|Z_A|}$$

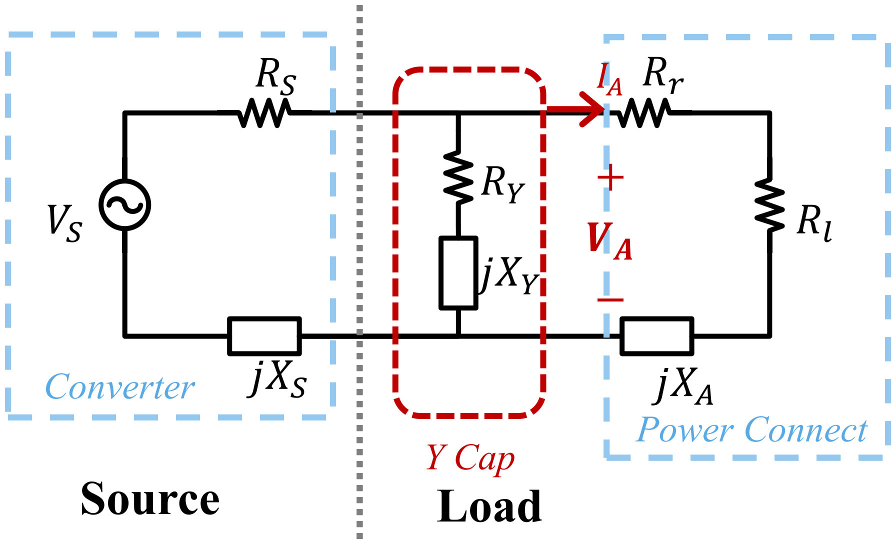
2. Around the resonant frequency of the antenna, because the reactance of the antenna is close to 0, $|Z_Y|$ may be larger than $|Z_A|$ in a narrow frequency range.

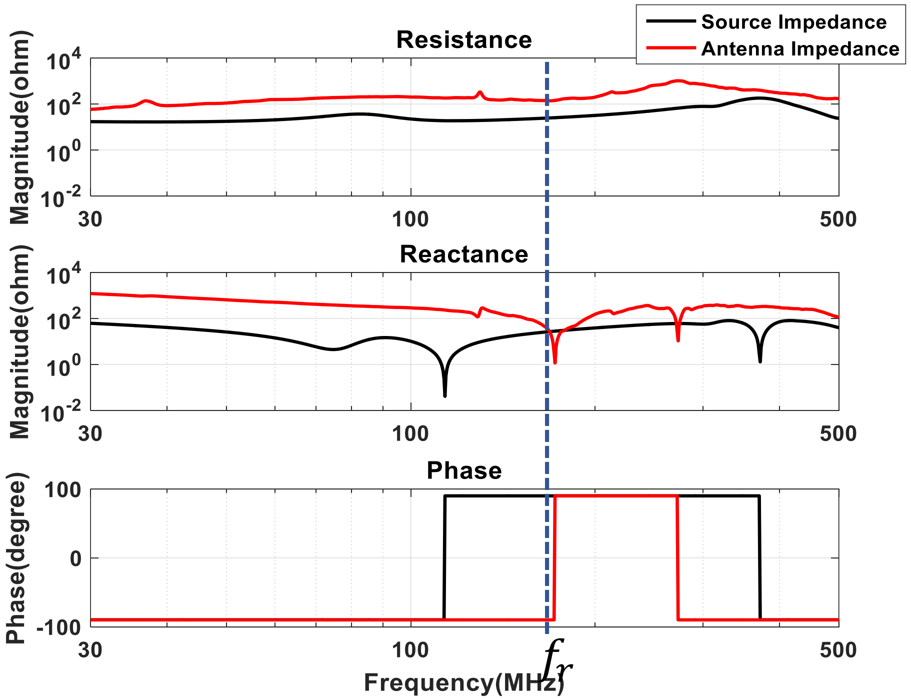
➔ $E_{max} = \sqrt{\frac{\eta G_o}{4\pi r^2}} \times |V_S| \times \left|\frac{Z_A}{Z_A+Z_S}\right| \times \frac{\sqrt{R_A}}{|Z_A|}$

$$= \sqrt{\frac{\eta G_o}{4\pi r^2}} \times |V_S| \times \frac{\sqrt{R_A}}{\sqrt{(R_S + R_A)^2+(X_S + X_A)^2}}$$

➔ E_{max} is the same as the one without Y cap.

Simplify the Equations of Radiated EMI





Usually, the parasitic resistance of the Y cap R_Y is smaller than R_S and $|X_Y|$

$$E_{max} = \sqrt{\frac{\eta G_o}{4\pi r^2}} \times |V_S| \times \frac{|X_Y|}{\sqrt{{R_S}^2 + (X_Y + X_S)^2}} \times \frac{\sqrt{R_A}}{\sqrt{{R_A}^2 + {X_A}^2}}$$

$$|I_A| = |V_S| \times \frac{|X_Y|}{\sqrt{{R_S}^2 + (X_Y + X_S)^2}} \times \frac{1}{\sqrt{{R_A}^2 + {X_A}^2}}$$

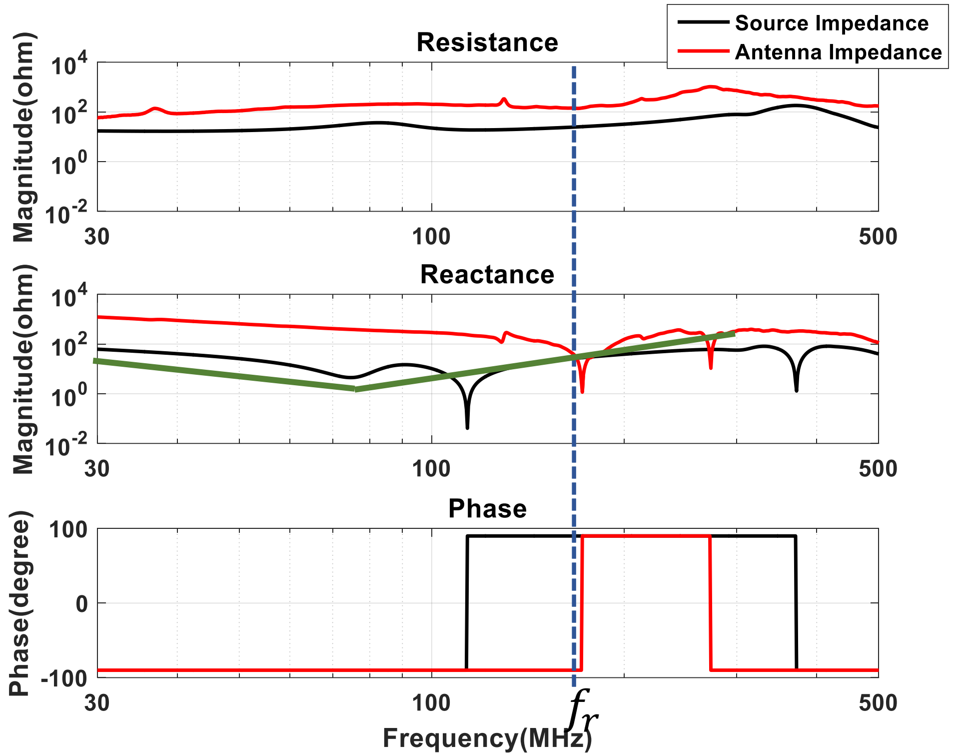
Around 30MHz

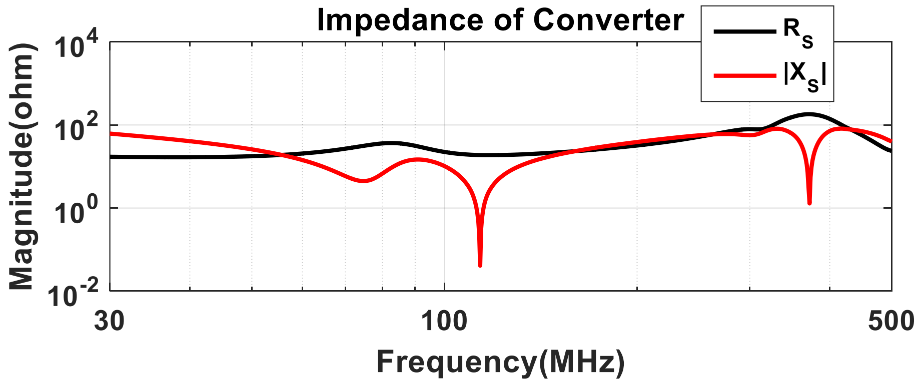
$$R_A \ll |X_A| \qquad R_S \ll |X_A| \qquad |X_S| \ll |X_A|$$

$$E_{max} = \sqrt{\frac{\eta G_o}{4\pi r^2}} \times |V_S| \times \frac{|X_Y|}{\sqrt{{R_S}^2 + (X_Y + X_S)^2}} \times \frac{\sqrt{R_A}}{|X_A|}$$

Insertion Gain$= \frac{|X_Y|}{\sqrt{{R_S}^2 + (X_Y + X_S)^2}}$

CM Capacitor Impedance Design around 30MHz





Insertion Gain= $\dfrac{|X_Y|}{\sqrt{R_S{}^2+(X_Y+X_S)^2}}$

(1) $|X_Y| < |X_S|$

Insertion Gain= $\dfrac{|X_Y|}{\sqrt{R_S{}^2+X_S{}^2}}$

$|X_Y| \downarrow$ ➔ Insertion Gain $\downarrow$

$|X_Y| < |X_S|$=62 around 30MHz

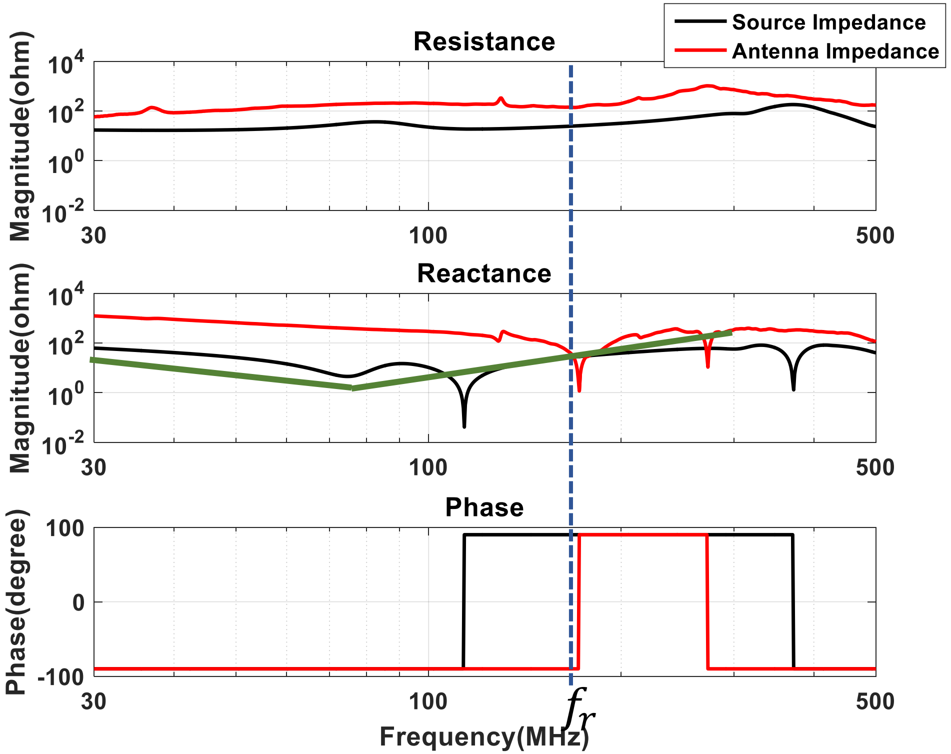
If X_Y is capacitive, $C > 85.6pF$
If X_Y is inductive, $L < 328.76nH$

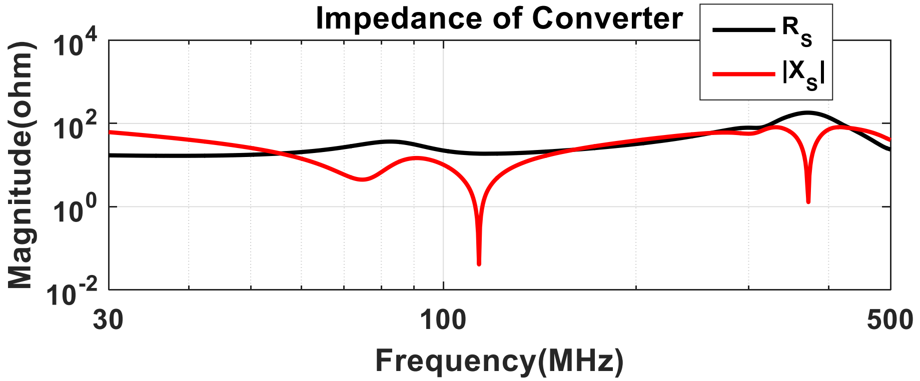
(2) $|X_Y| > |X_S|$

Insertion Gain= $\dfrac{|X_Y|}{\sqrt{R_S{}^2+X_Y{}^2}} = \dfrac{|X_Y|}{|X_Y|} = 1$

The Y cap can reduce the radiation only if its impedance is smaller than that of the converter.
And smaller impedance is more effective in this range.

CM Capacitor Impedance Design Around f_r





$$E_{max} = \sqrt{\frac{\eta G_o}{4\pi r^2}} \times |V_S| \times \frac{|X_Y|}{\sqrt{{R_S}^2 + (X_Y + X_S)^2}} \times \frac{\sqrt{R_A}}{\sqrt{{R_A}^2 + {X_A}^2}}$$

$$E_{\max\_old} = \sqrt{\frac{\eta G_o}{4\pi r^2}} \times |V_S| \times \frac{\sqrt{R_A}}{\sqrt{(R_S + R_A)^2 + (X_S + X_A)^2}}$$

Around the original resonant frequency f_r

$$X_S = -X_A,\ X_A < R_A,\ R_S < R_A$$

$$E_{\max\_old} = \sqrt{\frac{\eta G_o}{4\pi r^2}} \times |V_S| \times \frac{\sqrt{R_A}}{R_A}$$

$$E_{\max\_new} = \sqrt{\frac{\eta G_o}{4\pi r^2}} \times |V_S| \times \frac{|X_Y|}{\sqrt{{R_S}^2 + (X_Y + X_S)^2}} \times \frac{\sqrt{R_A}}{R_A}$$

Insertion Gain= $\frac{|X_Y|}{\sqrt{{R_S}^2 + (X_Y + X_S)^2}}$

Around f_r, R_S is close to X_S

Insertion Gain<1 ➔ $|X_Y| < |X_S| = 32.13$

If X_Y is capacitive, $C > 29.67pF$

If X_Y is inductive, $L < 30.62nH$

CM Capacitors with different Impedances

Around 30MHz: <62

If X_Y is capacitive, $C > 85.6pF$

If X_Y is inductive, $L < 328.76nH$

Around f_r: < 32.13

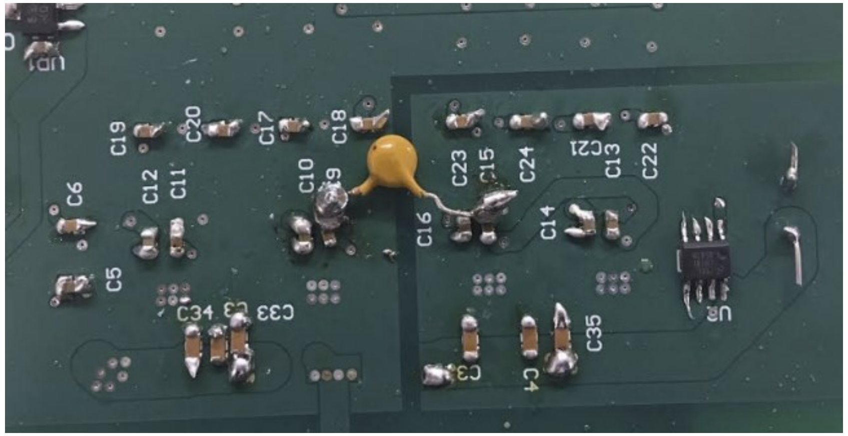
If X_Y is capacitive, $C > 29.67pF$

If X_Y is inductive, $L < 30.62nH$

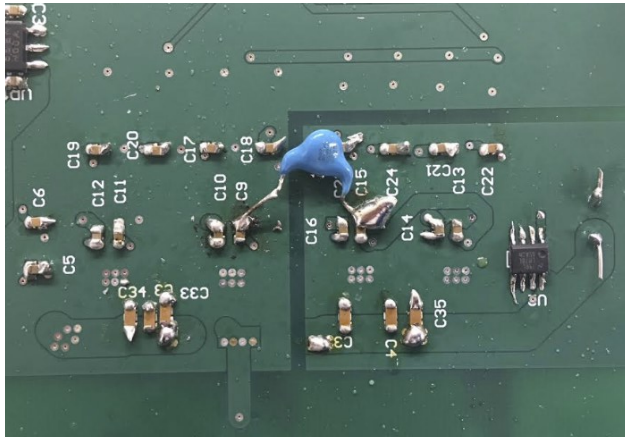
Both 30MHz and f_r:

If X_Y is capacitive, $C > 85.6pF$

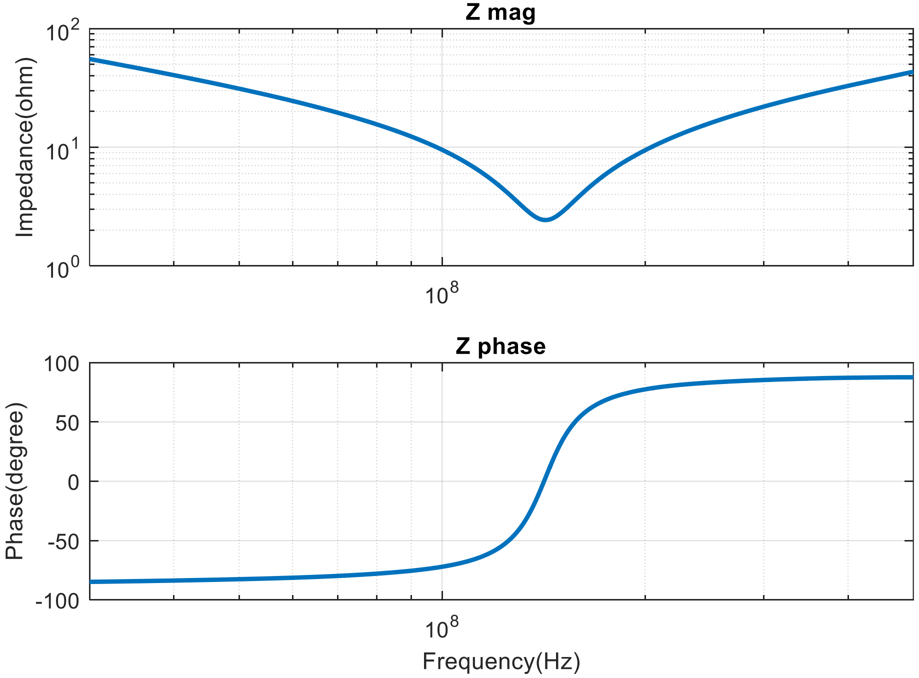
If X_Y is inductive, $L < 30.62nH$

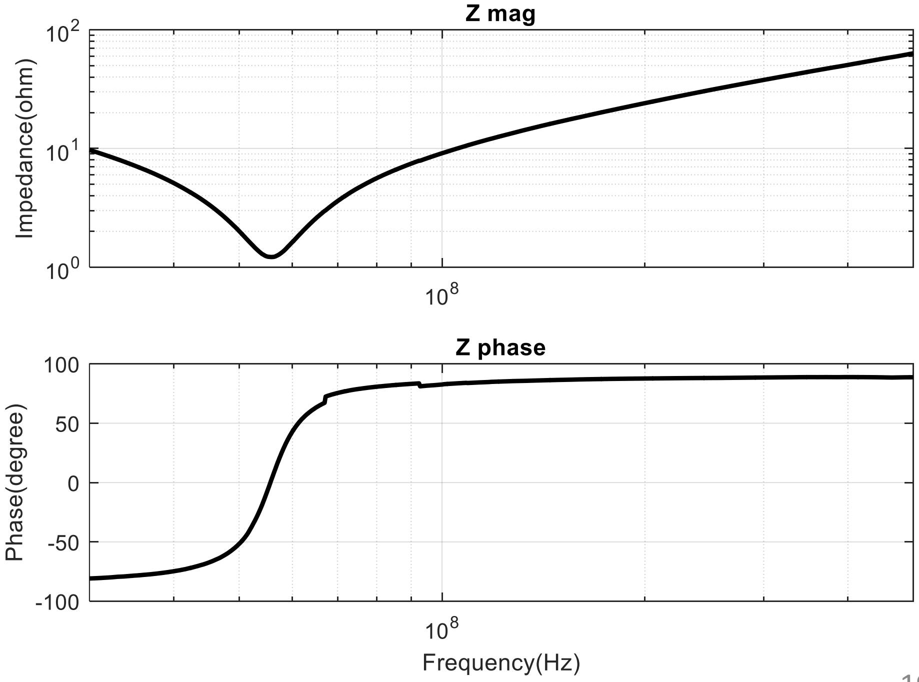


100pF

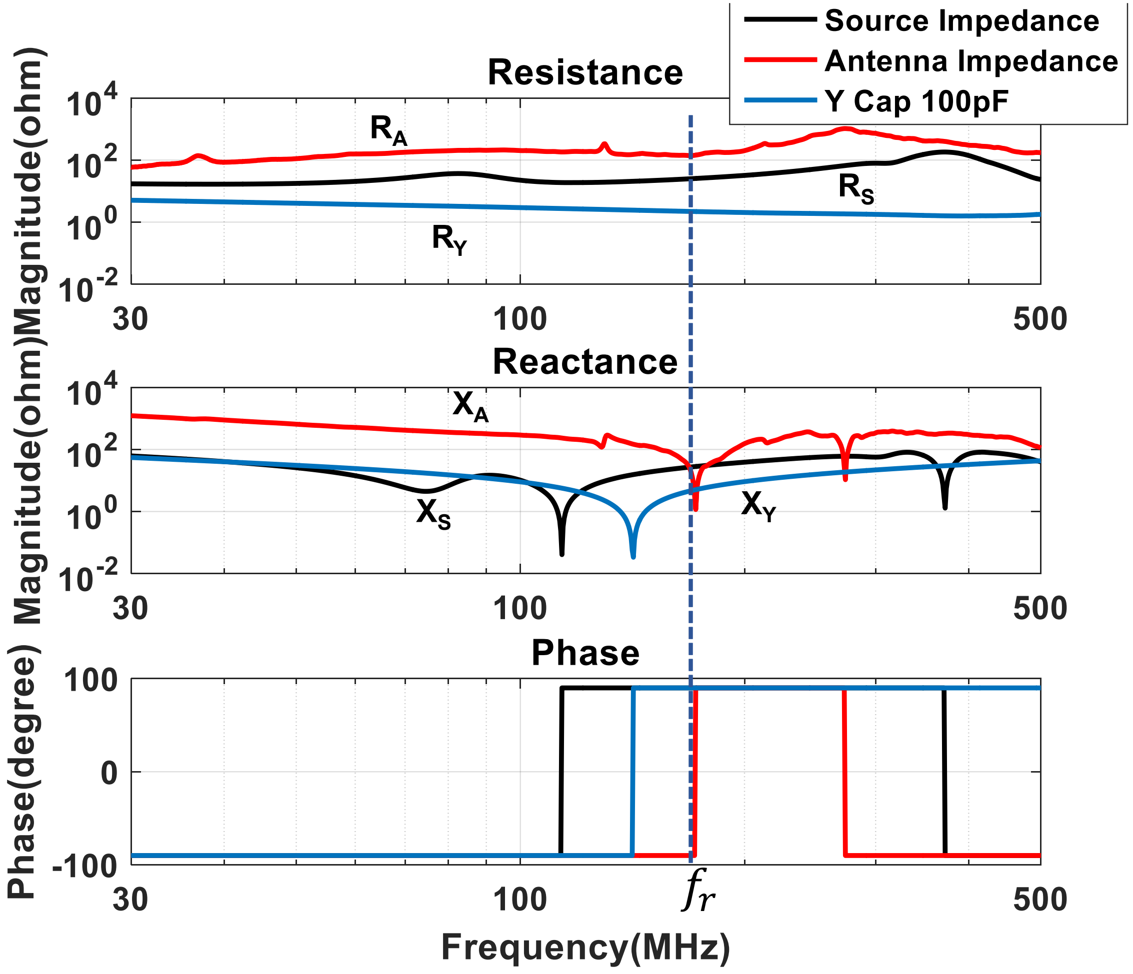


470pF





Performance of the 100pF CM Capacitor



1. Around 30MHz

$X_Y \approx X_S,\ R_S < |X_S|$

Insertion Gain= $\frac{|X_Y|}{\sqrt{{R_S}^2+(X_Y+X_S)^2}} = \frac{1}{2}$

The current/radiated E field will be reduced by 6dB.

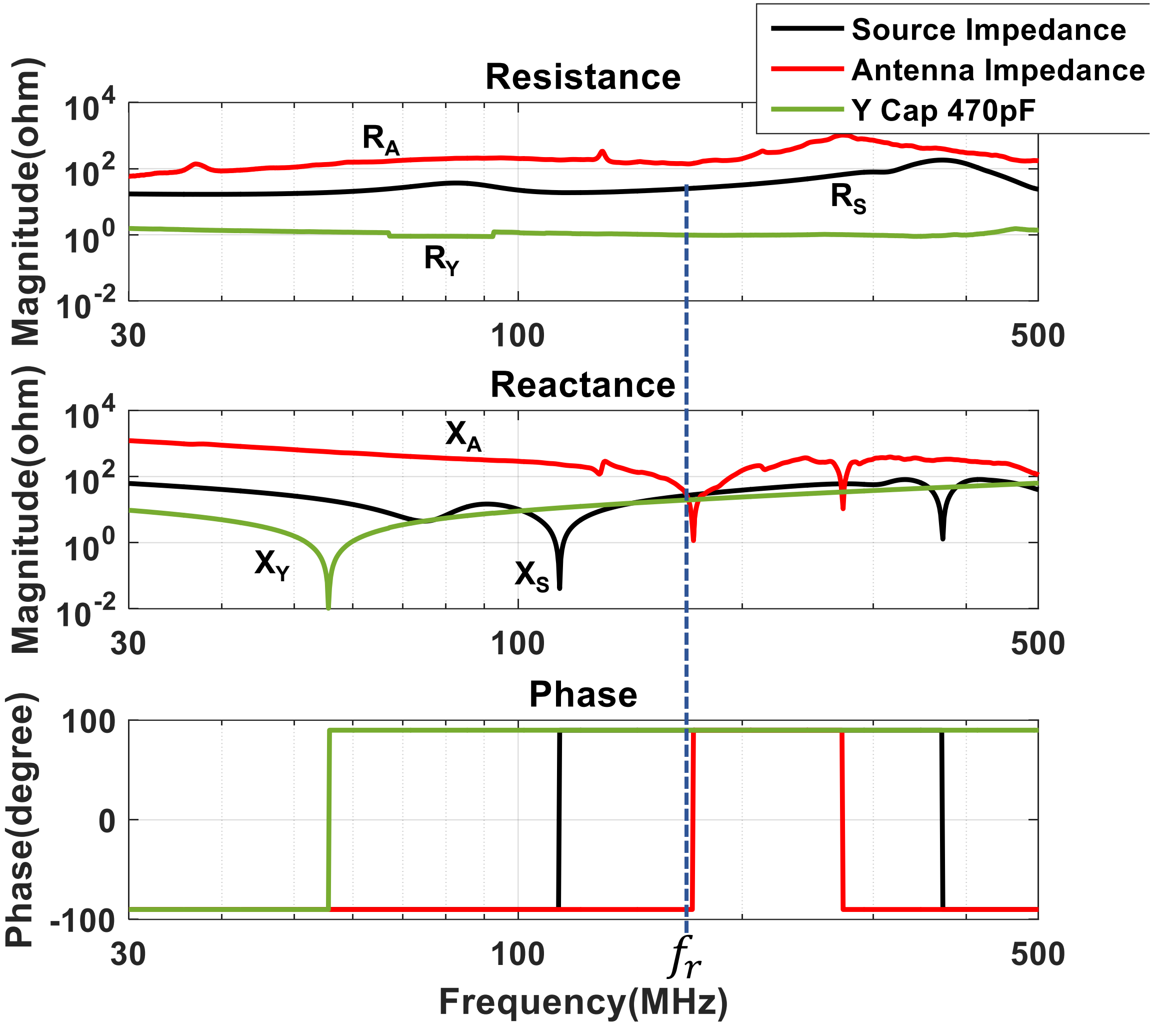
2. Around 167MHz(original resonant frequency f_r)

$$|X_Y| < |X_S|$$

The current/ radiated E field can be greatly reduced.

Insertion Gain= $\frac{|\boldsymbol{X_Y}|}{\sqrt{{\boldsymbol{R_S}}^2+(\boldsymbol{X_Y}+\boldsymbol{X_S})^2}}$

Performance of the 470pF CM Capacitor



1. Around 30MHz

$$|X_Y| < |X_S|$$

Current/radiated E field can be greatly reduced

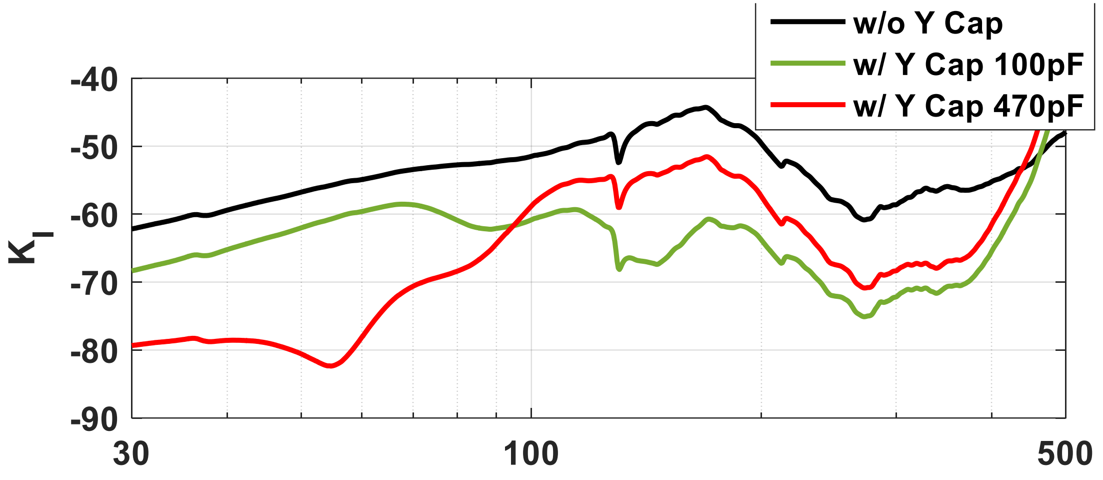
2. Around 167MHz

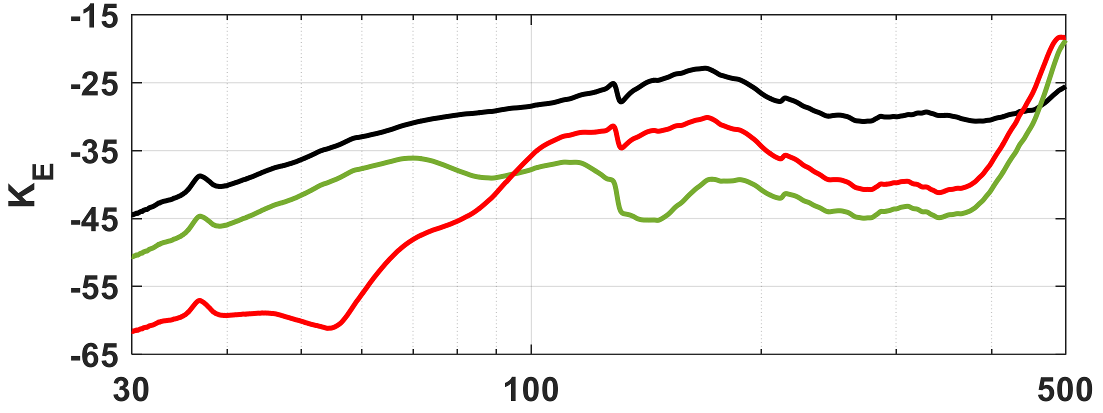
$$|X_Y| \approx |X_S|, |X_Y + X_S| > |R_S|$$

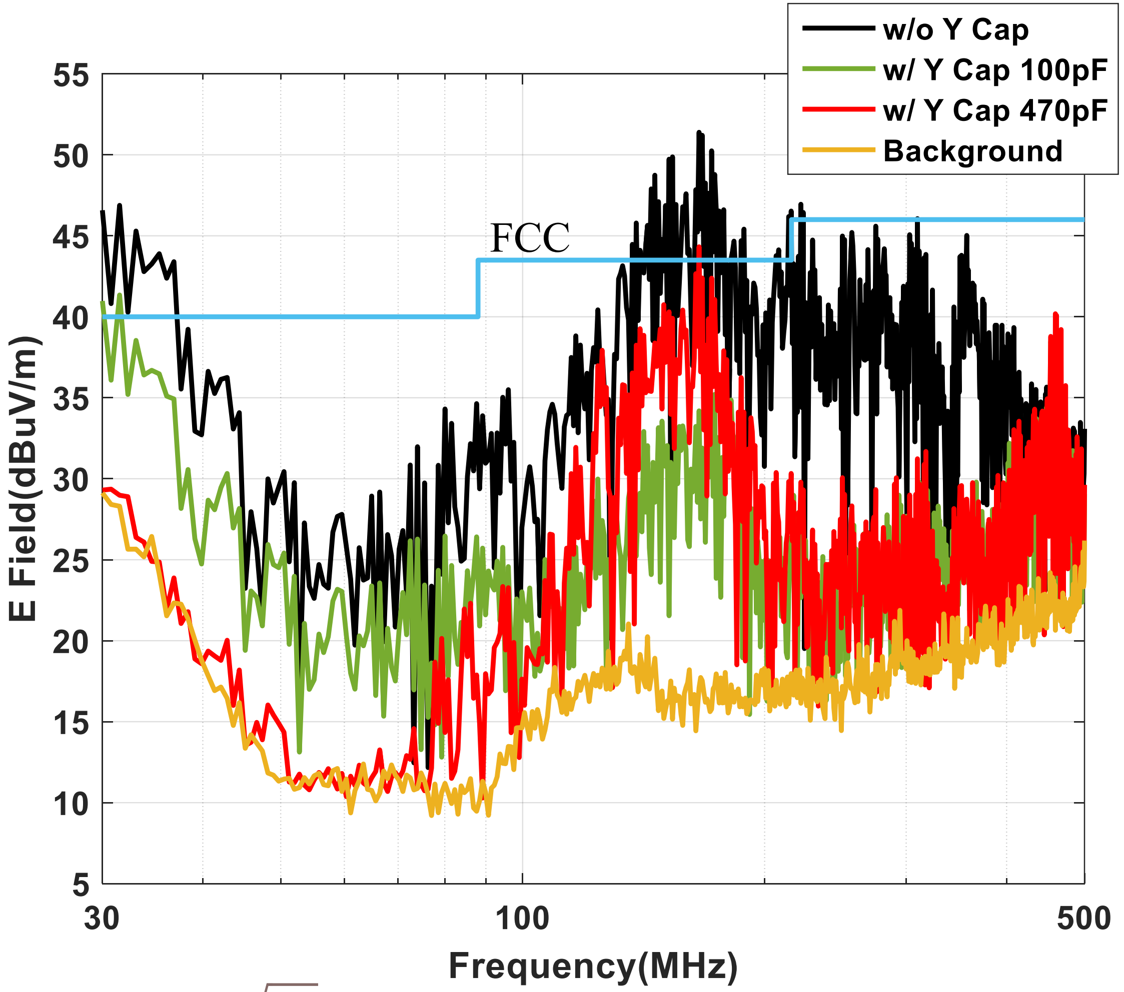
Insertion Gain= $\frac{|X_Y|}{\sqrt{R_S{}^2+(X_Y+X_S)^2}}$

The reduction about 6dB.

Experimental Verification





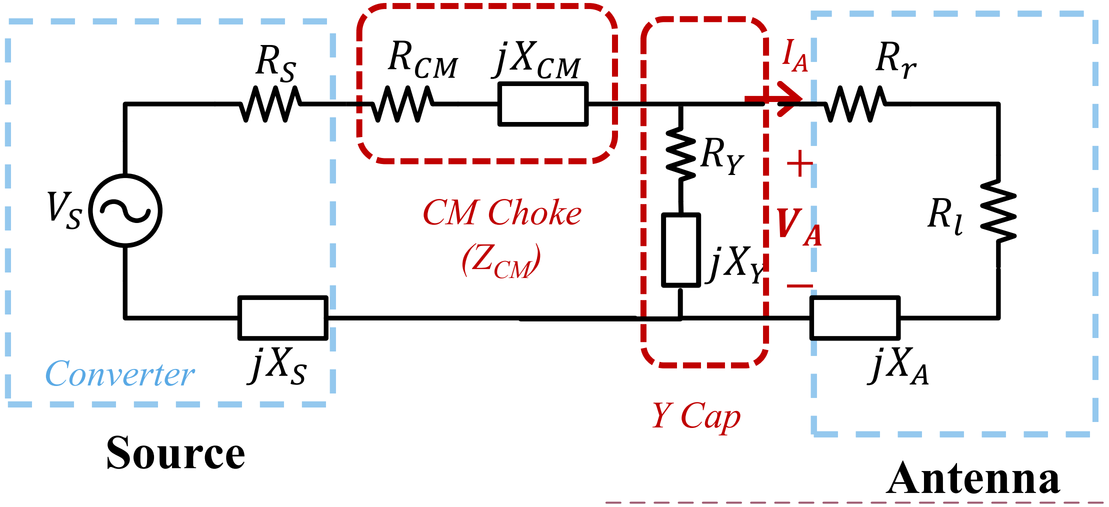


w/ CM capacitor:

$$K_I = \frac{|X_Y|}{\sqrt{{R_S}^2 + (X_Y + X_S)^2}} \times \frac{1}{\sqrt{{R_A}^2 + {X_A}^2}}$$

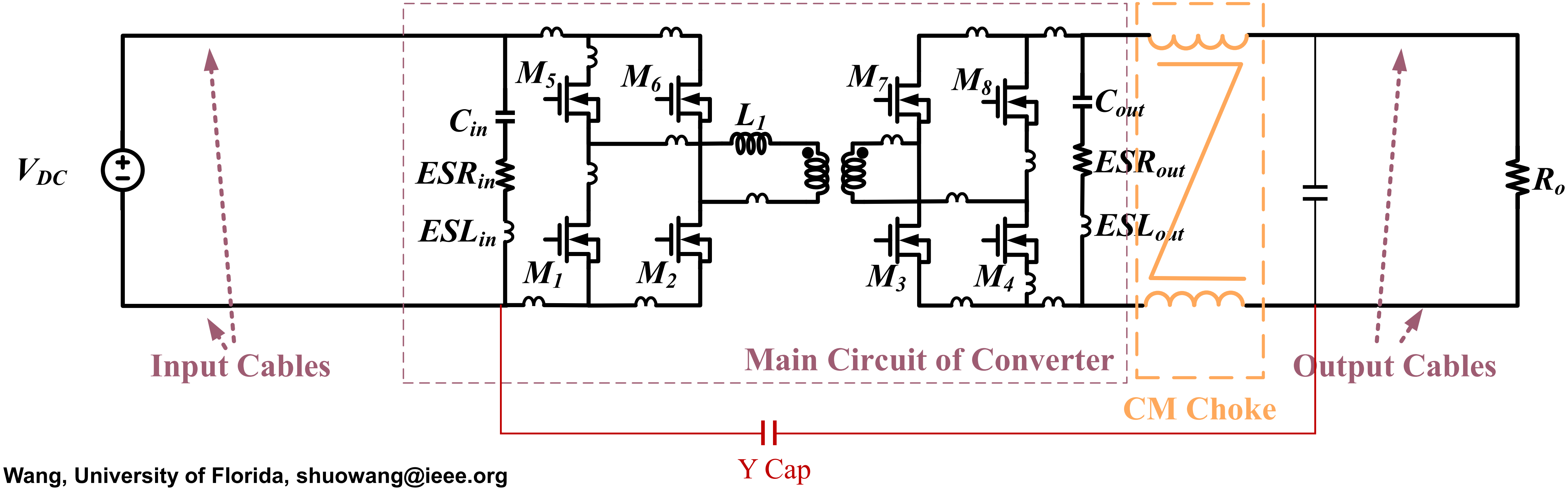
$$K_E = \frac{|X_Y|}{\sqrt{{R_S}^2 + (X_Y + X_S)^2}} \times \frac{\sqrt{R_A}}{\sqrt{{R_A}^2 + {X_A}^2}}$$

LC Type EMI Filter Design

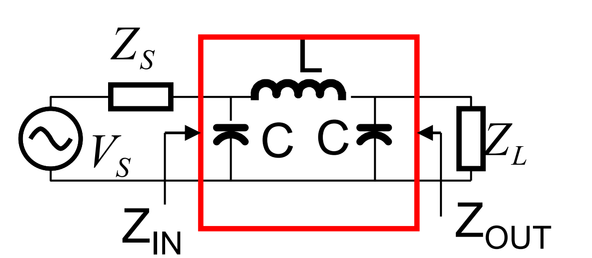
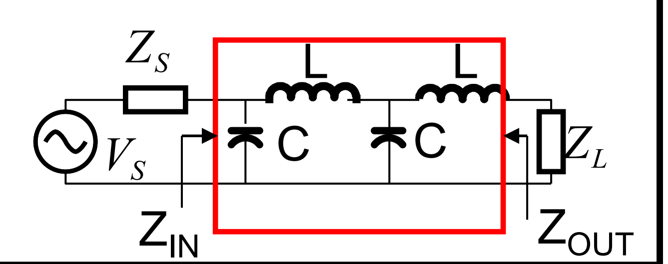
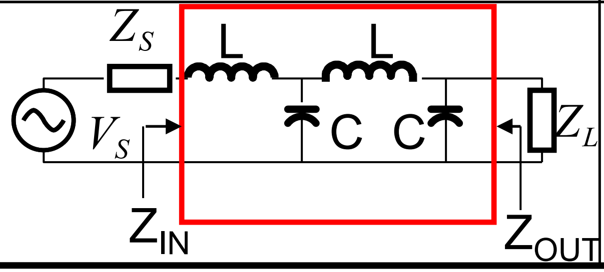
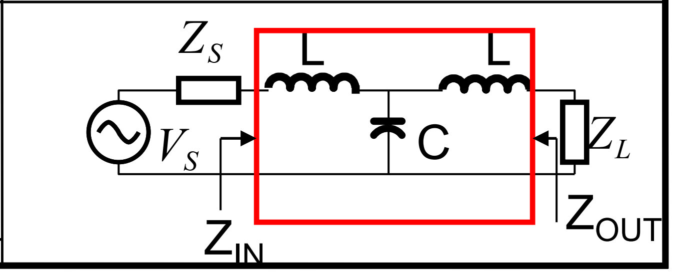


$$I_A = |V_S| \times \left| \frac{Z_Y \parallel Z_A}{Z_{CM} + Z_S + Z_Y \parallel Z_A} \times \frac{1}{Z_A} \right|$$

$$E_{\max} = \sqrt{\frac{\eta G}{4\pi r^2}} \times |V_S| \times \left| \frac{Z_Y \parallel Z_A}{Z_{CM} + Z_S + Z_Y \parallel Z_A} \times \frac{1}{Z_A} \right| \times \sqrt{R_A}$$

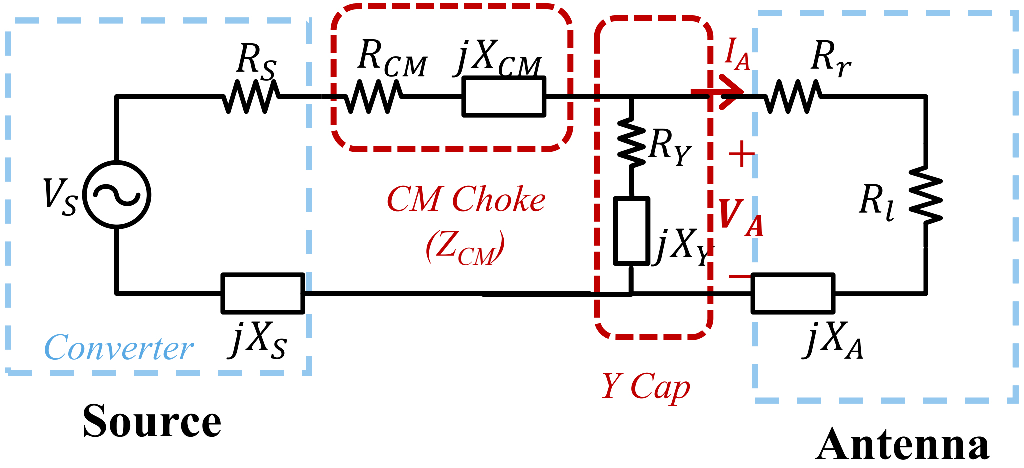


Filter Impedance Mismatch Design

$\lvert Z_S\rvert$ \ $\lvert Z_L\rvert$	High Z_L	Low Z_L
High Z_S		
Low Z_S		

$$I_A = \lvert V_S\rvert \times \left| \frac{Z_Y \parallel Z_A}{Z_{CM} + Z_S + Z_Y \parallel Z_A} \times \frac{1}{Z_A} \right|$$

$$E_{\max} = \sqrt{\frac{\eta G}{4\pi r^2}} \times \lvert V_S\rvert \times \left| \frac{Z_Y \parallel Z_A}{Z_{CM} + Z_S + Z_Y \parallel Z_A} \times \frac{1}{Z_A} \right| \times \sqrt{R_A}$$



At new resonant frequency $f_{r\_new\_LC}$:

$$I_A = \lvert V_S\rvert \times \frac{\lvert X_Y\rvert}{R_{CM} + R_S} \times \frac{1}{\sqrt{R_A^2 + X_A^2}}$$

$$E_{\max} = \sqrt{\frac{\eta G}{4\pi r^2}} \times \lvert V_S\rvert \times \frac{\lvert X_Y\rvert}{R_{CM} + R_S} \times \sqrt{K_E}$$

Make f_{CM} close to $f_{r\_new\_LC}$

$$R_{CM} = \begin{cases} \omega^2 \dfrac{L_{CM}^2}{R_p} & f \ll f_{CM} \\ R_p & f = f_{CM} \\ \dfrac{1}{\omega^2 C_p^2 R_p} & f \gg f_{CM} \end{cases}$$

$\lvert Z_{CM}\rvert \gg \lvert Z_S\rvert$ and $\lvert Z_Y\rvert \ll \lvert Z_A\rvert$ should be met.

$\lvert X_{CM}\rvert > \lvert X_S\rvert$, & $\lvert X_Y\rvert < \lvert X_A\rvert$ to reduce EMI at f_r.

Questions?